To be a Value of a Plural Variable, You Don't Have to be Plural (You Just Have to Be)

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Given a language with truth-evaluable *sentences that contain variables*, we say that such sentences are true or false *relative to assignments* of values to variables. Given a first-order language, with no plural variables, we say that each assignment A assigns *exactly one* value to each variable; where each value of a variable is an entity in the relevant domain. For example, the "open" sentence *Fx & Gx* has a singular variable, whose semantic role can be made explicit.

F( ) & *G*( ) is true relative to A iff **the value** that A assigns to *x* is such that:
|______| **it** satisfies *F*, and **it** satisfies *G*
x **A(*x*)** **A(*x*)**

Given plural variables, corresponding to 'them' (as opposed to 'it'), we can

(a) retain the idea that each assignment assigns *exactly one* value to each variable,
by saying that each value of a plural variable is a plural entity *with elements*
that can be values of singular variables; [standard view]

or (b) retain the idea that each value of a variable (singular or plural) is an entity
in the domain over which all variables range, by saying that a plural variable can have
many values relative to a single assignment of values to variables. [Boolos' view]

Φ( ) & Ψ( ) is true relative to A iff ...
|______|
x_{+pl}

(a) **the plural entity** that A assigns to x_{+pl} is such that: **it** satisfies Φ, and **it** satisfies Ψ
(b) **the entities** that A assigns to x_{+pl} are such that: **they** satisfy Φ, and **they** satisfy Ψ

CLAIMS: the second option is coherent, not a notational variant of the first, and preferable;
it can be part of a simple account of semantic composition that helps explain
some otherwise puzzling facts, including the conservativity of determiners

Assume a domain of exactly 5 things—a, b, c, d, e—and so 31 possible assignments to a variable

null	a	b	ba	c	ca	cb	cba
d	da	db	**dba**	dc	dca	dcb	dcba
e	ea	eb	eba	ec	eca	ecb	ecba
ed	eda	edb	edba	edc	edca	edcb	edcba

It might seem that mereology is the only natural construal of the lattice. But we need not assume one value per variable.

00000	00001	00010	00011	00100	00101	00110	00111
01000	01001	01010	**01011**	01100	01101	01110	01111
10000	10001	10010	10011	10100	10101	10110	10111
11000	11001	11010	11011	11100	11101	11110	11111

e=10000, d=1000, c=100, b=10, a=1
01011 = 1000 + 10 + 1 = d + b + a
which entit<u>ies</u> are assigned?
(e, ⊥), (**d**, ⊤), (c, ⊥), (**b**, ⊤), (**a**, ⊤)

A Common Interpretation of Indices, Plural Demonstratives, and Verb Phrases

(1) This trumps that — The sentence [This$_1$ [trumps that$_2$]] is true,
7♡ Q♧ — relative to an assignment A of values to variables, iff
A(1) trumps A(2); where for each index *i*,
A(*i*) is **the** entity that A assigns to the *i*th variable

(2) This trumps them — [This$_1$ [trumps them$_2$]] is true relative to A iff
7♡ Q♧ K◊ J♧ — $A(1)\ \text{trumps*}\ A(2)\ \&\ \neg\text{Plural}[A(1)]\ \&\ \text{Plural}[A(2)]$

(3) They trump it — [They$_1$ [trump it$_2$]] is true relative to A iff
7♡ 9♡ Q♧ — $A(1)\ \text{trumps*}\ A(2)\ \&\ \text{Plural}[A(1)]\ \&\ \neg\text{Plural}[A(2)]$

(4) They trump them — [They$_1$ [trump them$_2$]] is true relative to A iff
7♡ 9♡ Q♧ K◊ J♧ — $A(1)\ \text{trumps*}\ A(2)\ \&\ \text{Plural}[A(1)]\ \&\ \text{Plural}[A(2)]$

(5) for each entity, it is Plural iff it has other entities as elements

(6) $\forall x\{\text{Plural}(x) \rightarrow \exists y\exists z[(y \neq z)\ \&\ (y \in x)\ \&\ (z \in x)]\}$

(6a) $\forall x{:}\text{Plural}(x)\{\exists y\exists z[(y \neq z)\ \&\ (y \in x)\ \&\ (z \in x)]\}$

(6b) $\forall X\exists x\exists y[(x \neq y)\ \&\ (x \in X)\ \&\ (y \in X)]$ — $X/x{:}\text{Plural}(x)/x_{+pl}$

(7) for every plural entity X, plural entity Y, nonplural entity x, and nonplural entity y:
X trumps* Y iff every element of X trumps every element of Y,
X trumps* y iff every element of X trumps y,
x trumps* Y iff x trumps every element of Y, and
x trumps* y iff x trumps y

(8) $\forall X\forall Y\forall x\forall y\langle\ \{\text{Trumps*}(X, Y) \leftrightarrow \forall x'{:}x'{\in}X[\forall y'{:}y'{\in}Y\{\text{Trumps}(x', y')\}]\}\ \&$
$\{\text{Trumps*}(X, y) \leftrightarrow \forall x'{:}x'{\in}X[\text{Trumps}(x', y)]\}\ \&$
$\{\text{Trumps*}(x, Y) \leftrightarrow \forall y'{:}y'{\in}Y[\text{Trumps}(x, y')]\}\ \&$
$\{\text{Trumps*}(x, y) \leftrightarrow \text{Trumps}(x, y)]\}\ \rangle$

(9) This trumps that — [This$_1$ [trumps that$_2$]] is true relative to A iff
7♡ Q♧ — $A(1)\ \text{trumps*}\ A(2)\ \&\ \neg\text{Plural}[A(1)]\ \&\ \neg\text{Plural}[A(2)]$

(10) [__$_1$ [trump(s) __$_2$]] is true relative to A iff A(1) trumps* A(2)

(11) $\|\textit{trump(s)}\|^{A} = \lambda\beta.\lambda\alpha.\text{Trumps*}(\alpha, \beta)$ — *using number-neutral variables*

(12) Every heart trumps every club — $\forall x{:}\text{Heart}(x)\{\forall y{:}\text{Club}(y)[\text{Trumps*}(x, y)]\}$

(13) The hearts trump the clubs — $\iota X{:}\text{Hearts}(X)\{\iota Y{:}\text{Clubs}(Y)\{\text{Trumps*}(X, Y)\}\}$
$\exists X{:}[\forall x{:}x{\in}X \leftrightarrow \text{Heart}(x)]\{\exists Y{:}\forall y[y{\in}Y \leftrightarrow \text{Club}(y)]\{\text{Trumps*}(X, Y)\}\}$

(14) They$_1$ trump them$_2$ — $\text{Plural}[A(1)]\ \&\ \text{Plural}[A(2)]\ \&\ A(1)\ \text{trumps*}\ A(2)$
$\exists X{:}[\forall x{:}x{\in}X \leftrightarrow x{\in}A(1)]\{\exists Y{:}\forall y[y{\in}Y \leftrightarrow y{\in}A(2)]\{\text{Trumps*}(X, Y)\}\}$

Imagine a (team) game in which no one card trumps anything, but any 2 hearts trump any 2 clubs

(15) They$_1$ trump them$_2$ collective: Plural[A(1)] & Plural[A(2)] & A(1) trumps^{co} A(2)
7♡ 9♡ Q♧ J♧ distributive: Plural[A(1)] & Plural[A(2)] & A(1) trumps* A(2)

(16) for every plural entity X, and every plural entity Y, X trumps^{co} Y iff
the element<u>s</u> of X (together) trum<u>*p*</u> the element<u>*s*</u> of Y

QUESTIONS: is this just to say that X trumps^{co} Y iff X trumps Y?
Does {7♡, 9♡} trump {Q♧, J♧}? Or is 'trumps^{co}' a theoretical term we must define?
If we adopt the hypothesis that each plural demonstrative has a plural entity as its value, relative to each assignment of values to variables, what *else* do we need to say?
Are we forced to introduce *multiple type-shifting* principles for verb meanings?
[see Landman]

(17) They$_1$ wrote them$_2$ Plural[A(1)] & Plural[A(2)] & A(1) wrote^{co} A(2)
☺☹✍♀♂ ✉✉✉ ✉✉✉
a b c d e f g h i j k

(18) Five professors wrote six papers
(19) ∃X∃Y[FiveProfessors(X) & SixPapers(Y) & Wrote^{co}(X, Y)]
FiveProfessors(X) <–> FiveMembered(X) & ∀x:x∈X[Professor(x)]
SixPapers(X) <–> SixMembered(X) & ∀x:x∈X[Paper(x)]

total autonomy	*semi-cooperation*			*total cooperation*
a: f,g	a,b,c: f,g,h	a,b: f	...	a,b,c,d,e: f,g,h,i,j,k
b: h	d,e: i,j,k	c,b: g		
c: i		d,c: h		
d: j		a,c,e: i,j		
e: k		b,d: k		[see Gillon, Schein]

(20) Wrote^{co}({a, b, c, d, e}, {f, g, h, i, j, k})
if this allows for less than total cooperation, then what does it mean, if not:
the element<u>s</u> of {a, b, c, d, e} w<u>ere</u>, somehow, the writer<u>s</u> of the element<u>s</u> of {f, g, h, i, j, k}?

(21) Five professors wrote six papers in March (quickly, under pressure, and inelegantly)

(22) ∃X∃Y∃e{Agent(e, X) & |X| = 5 & ∀x:x∈X[Professor(x)] & PastWriting(e)
Theme(e, Y) & |Y| = 6 & ∀y:y∈Y[Paper(y)] & In(e, March) & ... }
<u>one</u> big event?

(23) ∃X∃Y∃E{Agent(E, X) & ... & ∀e:e∈E[PastWriting(e)] & Theme(E, Y) & ... }

each-of-X was-an-Agent-of some-of-E

each-of-E was-done-by some-of-X

each-of-E was-a-production-of some-of-Y

each-of-Y was-a-Theme-of some-of-E

A Simpler Alternative (see Boolos, Schein): let a variable have values relative to an assignment

(24) Five professors wrote six papers in March

$\exists$E{$\exists$X:FIVE(X) & Professors(X)[Agent(E, X)] & PastWriting(E) &
$\exists$X:SIX(X) & Papers(X)[Theme(E, X)] & In(E, March)}

There *are one or more things*$_E$ (the Es) such that:
(i) five professors were *their*$_E$ Agents—i.e.,
their$_E$ Agents were some things$_X$ such that: they$_X$ are five and they$_X$ are professors
and (ii) *they*$_E$ were events of writing—i.e.,
each of them$_E$ was an event of writing $\forall$e:Ee[PastWriting(e)]
and (iii) six papers were *their*$_E$ Themes—i.e.,
their$_E$ Themes were some things$_X$ such that: they$_X$ are six and they$_X$ are papers
and (iv) *they*$_E$ were in March—i.e.,
each of them$_E$ occurred in March

'Xx' *means* that x is an X—i.e., x is one of the Xs—*not* that x is an *element of* X

Professors(X)	the Xs are professors	$\forall$x:Xx[Professor(x)]
Papers(X)	the Xs are papers	$\forall$x:Xx[Paper(x)]
SIX(X)	the Xs are six$_{EssPl}$	$\exists$Y$\exists$z{Five(Y) & ¬Yz & $\forall$x[Xx <–> Yx v (x = z)]}
FIVE(X)	the Xs are five$_{EssPl}$	$\exists$Y$\exists$z{Four(Y) & ¬Yz & $\forall$x[Xx <–> Yx v (x = z)]}
Agent(E, X)	the Xs are the Agents of the Es	$\forall$x:Xx{$\exists$e:Ee[Agent(e, x)]} & $\forall$e:Ee{$\exists$x:Xx[Agent(e, x)]}
Theme(E, X)	the Xs are the Themes of the Es	$\forall$x:Xx{$\exists$e:Ee[Theme(e, x)]} & $\forall$e:Ee{$\exists$x:Xx[Theme(e, x)]}

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(17) They$_1$ wrote them$_2$ TRUE relative to an assignment A iff
the things that A assigns to the first index *wrote*
(i.e., *were the Agents of some PastWritings whose Themes were*)
the things that A assigns to the second index

$\exists$E{$\exists$X:$\forall$x(Xx <–> Assigns(**A**, x, '1'))[Agent(E, X)] & PastWriting(E) &
$\exists$X:$\forall$x(Xx <–> Assigns(**A**, x, '2'))[Theme(E, X)]}

(18) Five professors wrote six papers *suitably neutral about cooperation*
$\exists$E{$\exists$X:Five(X) & Professors(X)[Agent(E, X)] & PastWriting(E) &
$\exists$X:Six(X) & Papers(X)[Theme(E, X)]}

a,b: 1 a,c,e: 45
c,b: 2 b,d: 6
d,c: 3

?
ϵ_1 ϵ_2 ϵ_3 ϵ_4 ϵ_5 ϵ_6
/ \ / \ / \ / | \ / | \ / \
ϵ_{1a} ϵ_{1b} ϵ_{2c} ϵ_{2b} ϵ_{3d} ϵ_{3c} ϵ_{4a} ϵ_{4c} ϵ_{4e} ϵ_{5a} ϵ_{5c} ϵ_{5e} ϵ_{6b} ϵ_{6d}

(25) The rocks rained down on the huts clustered near the lakes in which our ancestors fished

Another Familiar Theory and Some Further Familiar Questions

(26) **Every** bottle fell $\{z: \text{Fell}(z)\} \supseteq \{z: \text{Bottle}(z)\}$
the fallen **include** *the bottles*

$\|\text{every}\| = \lambda Y.\lambda X.\{z: X(z)\} \supseteq \{z: Y(z)\}$

$|\{z: Y(z)\} - \{z: X(z)\}| = 0$ $|\{z: \text{Bottle}(z)\} - \{z: \text{Fell}(z)\}| = 0$

$\lambda x.\text{Bottle}(x) = \|\text{bottle}\|$ $\|\text{fell}\| = \lambda x.\text{Fell}(x)$ *the bottles* **are among** *the fallen*

(27) **Most** bottles fell $|\{z: \text{Bottle}(z)\} \cap \{z: \text{Fell}(z)\}| > |\{z: \text{Bottle}(z)\} - \{z: \text{Fell}(z)\}|$
the bottles that fell **outnumber** *the bottles that didn't fall*

(28) Every bottle fell iff every bottle is a bottle that fell [see, *e.g.*, Barwise&Cooper]
Most bottles fell iff most bottles are bottles that fell
Some/No/The bottle(s) fell iff *some/no/the* bottle(s) izza/are bottle(s) that fell
Between five and eleven bottles fell iff *between five and eleven* bottles are bottles that fell

(29) [[DET NOUN] PREDICATE] *iff* [[DET NOUN] copula [NOUN that PREDICATE]]

(30) $\|\text{bottle(s) that fell}\| = \lambda x.\text{Bottle}(x) \,\&\, \text{Fell}(x)$

(31) Most bottles are bottles that fell
$|\{z: \text{Bottle}(z)\} \cap \{z: \text{Bottle}(z) \,\&\, \text{Fell}(z)\}| > |\{z: \text{Bottle}(z)\} - \{z: \text{Bottle}(z) \,\&\, \text{Fell}(z)\}|$
the bottles that are bottles that fell **outnumber** *the bottles that are not bottles that fell*

(32) The bottles **are equinumerous with** the things that fell
The bottles **"samenumber" (correspond 1-to-1 with)** the things that fell

(33) Equi bottles fell $|\{z: \text{Bottle}(z)\}| = |\{z: \text{Fell}(z)\}|$
$\|\text{Equi}\| = \lambda Y.\lambda X.\, |\{z: Y(z)\}| = |\{z: X(z)\}|$ *the bottles* **samenumber** *the fallen*

(34) Equi bottles are bottles that fell $|\{z: \text{Bottle}(z)\}| = |\{z: \text{Bottle}(z) \,\&\, \text{Fell}(z)\}|$
the bottles **samenumber** *the bottles that fell*

(35) Equi bottles fell iff equi bottles are bottles that fell *FALSE!*
$|\{z: \text{Bottle}(z)\}| = |\{z: \text{Fell}(z)\}|$ iff $|\{z: \text{Bottle}(z)\}| = |\{z: \text{Bottle}(z) \,\&\, \text{Fell}(z)\}|$

(36) The bottle fell *A bottle, and there was only one, fell*
$|\{z: \text{Bottle}(z)\}| = 1 \,\&\, |\{z: \text{Bottle}(z)\} \cap \{z: \text{Fell}(z)\}| > 0$

(37) Gre bottle fell *A bottle was the only thing that fell*
$|\{z: \text{Fell}(z)\}| = 1 \,\&\, |\{z: \text{Bottle}(z)\} \cap \{z: \text{Fell}(z)\}| > 0$

(38) Gre bottle is a bottle that fell *A bottle was the only bottle that fell*
$|\{z: \text{Bottle}(z) \,\&\, \text{Fell}(z)\}| = 1 \,\&\, |\{z: \text{Bottle}(z)\} \cap \{z: \text{Fell}(z)\}| > 0$

bottle-1	bottle-2	cup-1
fell	*didn't fall*	*fell*

(37) can be false, while (38) is true

Using Barwise and Cooper’s terminology: determiners “live on” their internal arguments; some but not all relations between functions (from individuals to truth values) are *conservative*:

$$X \,\mathbb{R}\, Y \qquad \text{iff} \qquad (X \cap_{<x,\mathbf{t}>} Y) \,\mathbb{R}\, Y$$

But many otherwise “natural” second-order relations, like equinumerosity, are nonconservative.
So why don’t we find determiners that—like the invented term ‘Equi’—express such relations?

Keenan and Stavi suggest that all determiner meanings are constructible, in a conservativity-preserving way, from “basic” determiner meanings that are conservative.
But even if this is right: *why* is the ‘Equi’-relation, which lies near the heart of arithmetic, not a basic determiner meaning? *Why* is ‘Most’ constructible, while ‘Equi’ is not?

Why is ‘The’ more natural than ‘Gre’? If determiners are of type <<x, **t**> , <<x, **t**>, **t**>>,

why are certain functions of this type *not* possible determiner meanings?

Why *don’t* we lexicalize ‘only’ as a determiner that is the nonconservative converse of ‘every’?

Compare the absence of “thematically inverted” verbs: $\|\text{grote}\| = \lambda y.\lambda x.y$ wrote x.

The invented phrases ‘Equi bottles’ and ‘Gre bottle’ would not be *restricted* quantifiers.
But this is another form of the question: if determiners express second-order relations, why do they express relations corresponding to restricted quantifiers (with the noun as restrictor)?

A related question, assuming that determiner phrases like ‘every bottle’ and ‘most bottles’ raise.

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        /    \                                /    \
  D(P)1        S                        D1           S              consider
               ...                                   ...
   /  \       /    \                    /  \        /    \          a familiar
  D    N    ...    V(P)                D    N     ...     V
Every bottle       /  \              Most bottles        /  \       idea about
                  V    t1                               V    t1
                 fell                                dropped        why they raise
```

Suppose that the lexical meaning of ‘every’ would not be properly expressed if ‘every bottle’ was interpreted as an argument of ‘fell’. If a determiner takes an internal and external argument, like a transitive verb, maybe ‘every’ raises to “see” its external argument and “express itself.” If so, this lexical requirement is satisfied in the configuration above—with the determiner taking a *sentential* external argument, whose value is TRUE or FALSE, relative to any assignment of values to variables. But does this fit with the idea that ‘every’ indicates a relation between *sets*? A sentence with one variable is, in many ways, *like* the corresponding predicate of type <x, **t**>. But if ‘every’ raises to a position in which its lexical requirements are met, why do we still have to “cheat” by construing the open *sentence* as a device for expressing a *function* of type <x, **t**>?

(39) $\langle \text{It}_1 \text{ fell}\rangle_S$ — TRUE, relative to any assignment A, iff A(1) fell

(39a) $1^\wedge\langle \text{t}_1 \text{ fell}\rangle_S$ — λx. (TRUE iff) x fell, relative to any assignment A

(40) $\langle \text{He}_1 \text{ dropped it}_2\rangle_S$ — TRUE, relative to any assignment A, iff A(1) dropped A(2)

(40) $2^\wedge\langle \text{He}_1 \text{ dropped it}_2\rangle_S$ — λx. (TRUE iff) A(1) dropped x, relative to any assignment A

(41) Every bottle [$\cancel{\text{wh}}_2$ $\langle \text{he}_1 \text{ dropped t}_2\rangle_S$] — has *no* truth-evaluable reading: why not?

(42) They$_1$ wrote them$_2$ ∃E[Agent(E, They$_1$) & PastWriting(E) & Theme(E, them$_2$)]

PastWriting(E) –> ∀e:Ee[Event(e)]
Event(e) –> ∀x{[External(e, x) <–> Agent(e, x)] & [Internal(e, x) <–> Theme(e, x)]}

(43) They$_1$ wrote them$_2$ ∃EExternal(E, They$_1$) & PastWriting(E) & Internal(E, them$_2$)]

(44) External(E, They$_1$) <–> ∃X:∀x[Xx <–> Assigns(A, x, '1')]{External(E, X)}
(45) Internal(E, them$_2$) <–> ∃X:∀x[Xx <–> Assigns(A, x, '2')]{Internal(E, X)}
ιX: Assigns(A, X, '2')

X X

∃{ι: Assigns(A, _ , '1')[Ext(_ , _)] & PastWriting(_) & ι: Assigns(A, _ , '2')[Int(_ , _)]}

E

(46) There were *one or more* things$_E$ such that
their$_E$ ExternalParticipant*s* (Agent*s*) were the thing*s* Assigned to the first index, <u>and</u>
they$_E$ were events of writing, <u>and</u>
their$_E$ InternalParticipant*s* (Theme*s*) were the thing*s* Assigned to the second index

(47) Every bottle fell [cp. Heim&Kratzer]

S
1 *EXT*
S ==> TRUE relative to A iff A(1) fell
Every *INT* ...
bottle
fell *int*
$t_{1:\neg pl}$ (singular variable)

∃{PastFalling(_) & Theme[_, A(1)]}
there was a falling of <u>the</u> thing
Assigned to the first index

(48) ∃{Every( ) & Internal[(_), Bottle( )] & 1^External[(_), TRUE relative to A iff A(1) fell)]}
F

PROPOSAL: determiners are predicates of "FregePairs," ordered pairs of the form <V, x>;
where the external element V is a truth value (TRUE or FALSE, ⊤ or ⊥), and
the internal element x is one of the things over which (singular and plural) variables range

Every(F) — *the Fs are all of the form* <⊤, x>
∀f:Ff[External(f, ⊤)]

Internal[F, bottle( )] — *the InternalParticipants of the Fs are the bottles*
ιX:Bottle(X)[Internal(F, X)]

1^External[F, TRUE relative to A iff A(1) fell] — *the Fs conform to the following rule: ⊤ iff x fell*
for each$_f$ F, its$_f$ ExternalParticipant is ⊤ iff
its$_f$ InternalParticipant fell

If the noun ‘bottle’ appears as the internal argument of a determiner, then (relative to any assignment A)
some FregePairs$_F$ (the Fs) are semantic values of that internal argument iff
their$_F$ InternalParticipants are the bottles

If the open sentence ⟨[fell $t_{1:\neg pl}$]⟩$_S$ appears as the external argument of an indexed determiner, then relative to any assignment A, some FregePairs$_F$ (the Fs) are semantic values of that external argument iff for each$_f$ of them$_F$: its$_f$ ExternalParticipant is ⊤ iff the open sentence is TRUE *relative to* the (minimal) variant of A that assigns its$_f$ InternalParticipant to the indexed variable

To illustrate, suppose a domain of exactly three bottles and two cups: b1, b2, b3, c1, c2

[Every bottle]$_D$ ⟨[fell __]⟩$_S$
|________|
i

(49) ∃F{Every(F) & Internal[F, bottle( )] & *i*^External[F, TRUE relative to A iff A(*i*) fell]}

The phrase ‘Every bottle’ imposes two conditions on semantic values$_F$ (the Fs):
Every one of them$_F$ must be of the form <⊤, x>, and
their$_F$ InternalParticipants must be (all and only) the bottles.
So there is only one “choice” of FregePairs that will satisfy the determiner phrase:
<⊤, b1>, <⊤, b2>, <⊤, b3>
These *three* FregePairs are (together) the semantic values of ‘Every bottle’.
Every bottle fell iff these FregePairs conform to the following rule: ⊤ iff x fell.

Some(F) ∃f:Ff[External(f, ⊤)]
No(F) ¬∃f:Ff[External(f, ⊤)]
Most(F) ∃Y∃N∀f{Outnumber(Y, N) & [Yf <–> Ff & External(f, ⊤)] & [Nf <–> Ff & External(f, ⊥)]}

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(50) Every bottle fell
∃F⟨EVERY(F) & ιX:Bottle(X)[Internal(F, X)] &
for each$_f$ F: its$_f$ ExternalParticpant is ⊤ iff its$_f$ InternalParticipant **fell**⟩

(51) Every bottle is a bottle that fell
∃F⟨EVERY(F) & ιX:Bottle(X)[Internal(F, X)] &
for each$_f$ F: its$_f$ ExternalParticpant is ⊤ iff its$_f$ InternalParticipant **is a bottle that fell**⟩

If the bottles are the InternalParticipants of the Fs, and f is an F whose InternalParticipant is x,
then *trivially*: [External(f, ⊤) <–> Fell(x)] *iff* [External(f, ⊤) <–> Bottle(x) & Fell(x)]
The “trick” is to *not* ignore the syntax: external arguments of determiners are *sentential*—
(assignment-relative) expressions of type <**t**>, *and not* disguised predicates of type <x, **t**>.
We don’t *need* variables ranging over functions to capture Frege’s insights about quantification.
We don’t *need* to associate arguments of determiners with extensions, and maybe we *shouldn’t*.

(52) [[Every bottle]$_D$ ⟨he dropped _ ⟩$_S$] (53) *[[Every bottle]$_D$ [~~wh~~ ⟨he dropped __ ⟩$_S$]]
|____________| |____________|

(54) There are many sets. None of them are selfelemental. But all of them are selfidentical.

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Appendix: Everybody Needs Event Variables

(1) Plum stabbed Green quickly with a knife
(2) Plum stabbed Green with a knife quickly
(3) Plum stabbed Green quickly
(4) Plum stabbed Green with a knife
(5) Plum stabbed Green

$$\begin{tikzcd} (1) \arrow[rr, leftrightarrow] \arrow[d] & & (2) \arrow[d] \\ (3) \arrow[r] & (5) & (4) \arrow[l] \end{tikzcd}$$

$(3)\ \&\ (4) \not\longrightarrow (1)$

see Davidson (1967, 1985), Taylor (1985), Parsons (1990), etc.

(1a) At least one stabbing of Green by Plum was done quickly and with a knife
(1b) $\exists e\{\text{PastStabOfGreenByPlum}(e)\ \&\ \text{Quick}(e)\ \&\ \text{With}(e, \text{a knife})]\}$
(1c) $\exists e\{\text{Agent}(e, \text{Plum})\ \&\ \text{PastStab}(e)\ \&\ \text{Theme}(e, \text{Green})\ \&\ \text{Quick}(e)\ \&\ \exists x{:}\text{Knife}(x)[\text{With}(e, x)]\}$

(6) $\exists x[\text{Red}(x)\ \&\ \text{Ball}(x)] \longrightarrow \exists x[\text{Red}(x)]\ \&\ \exists x[\text{Ball}(x)]$
(7) $\exists x[\text{Red}(x)]\ \&\ \exists x[\text{Ball}(x)] \not\longrightarrow \exists x[\text{Red}(x)\ \&\ \text{Ball}(x)]$

see Castañeda (1967), Carlson (1985), Higginbotham (1985)

(8) Plum kicked Green
(9) Green was kicked by Plum
(10) Green was kicked
(11) Plum kicked
(12) Plum kicked the ball
(13) Plum kicked the ball to Green
(14) Plum kicked Green the ball
(15) The ball was kicked
(16) Plum kicked to Green

$$\begin{tikzcd} (8) \arrow[rr, leftrightarrow] \arrow[d] & & (9) \arrow[d] \\ (11) & & (10) \\ (12) \arrow[u] \arrow[d] & (13) \arrow[l] \arrow[r, leftrightarrow] & (14) \\ (15) & & \end{tikzcd}$$

$(16) \longrightarrow (11)$

$(10)\ \&\ (11) \not\longrightarrow (8)$
$(15)\ \&\ (16) \not\longrightarrow (13)$
$(15)\ \&\ (12) \not\longrightarrow (13)$

(8a) $\exists e[\text{Agent}(\text{Plum})\ \&\ \text{PastKick}(e)\ \&\ \text{Theme}(e, \text{Green})]$
(9a) $\exists e[\text{Theme}(e, \text{Green})\ \&\ \text{PastKick}(e)\ \&\ \text{Agent}(\text{Plum})]$
(10a) $\exists e[\text{Theme}(e, \text{Green})\ \&\ \text{PastKick}(e)]$
(11a) $\exists e[\text{Agent}(e, \text{Plum})\ \&\ \text{PastKick}(e)]$
(12a) $\exists e[\text{Agent}(\text{Plum})\ \&\ \text{PastKick}(e)\ \&\ \text{Theme}(e, \text{the ball})]$
(13a) $\exists e[\text{Agent}(\text{Plum})\ \&\ \text{PastKick}(e)\ \&\ \text{Theme}(e, \text{the ball})\ \&\ \text{Goal}(e, \text{Green})]$
(14a) $\exists e[\text{Agent}(\text{Plum})\ \&\ \text{PastKick}(e)\ \&\ \text{Goal}(e, \text{Green})\ \&\ \text{Theme}(e, \text{the ball})]$
(15a) $\exists e[\text{Theme}(e, \text{the ball})\ \&\ \text{PastKick}(e)]$
(16a) $\exists e[\text{Agent}(e, \text{Plum})\ \&\ \text{PastKick}(e)\ \&\ \text{Goal}(e, \text{Green})]$

(17) On Monday, Plum hit Green the ball with a red stick
(18) On Tuesday, Plum hit the ball to Green with a blue stick
(19) On Wednesday, Plum hit the balls to Green with red sticks *see Schein (1993)*
(20) On Thursday, they hit twenty balls to them with blue sticks

(21) The senator called the millionaire from Texas *see Pietroski (2005)*
(a) The senator called the millionaire from Texas, and the millionaire was from Texas
(b) The senator called the millionaire from Texas, and the call was from Texas
#(c) The senator called the millionaire from Texas, and the senator was from Texas

(G) [[The senator] [called [the [millionaire [from Texas]]]]] (a)
(M) $\exists e\{\iota x{:}\text{Senator}(x)[\text{Agent}(e, x)]\ \&\ \text{PastCall}(e)\ \&\ \iota x{:}\text{Mill}(x)\&\text{From}(x, \text{Texas})[\text{Theme}(e, x)]\}$
(G') [[The senator] [[called [the millionaire]] [from Texas]]] (b)/#(c)
(M') $\exists e\{\iota x{:}\text{Senator}(x)[\text{Agent}(e, x)]\ \&\ \text{PastCall}(e)\ \&\ \iota x{:}\text{Mill}(x)[\text{Theme}(e, x)]\ \&\ \text{From}(e, \text{Texas})\}$