Solutions

1 Allocation of Public Spending

The government allocates a fixed budget g across two groups, only one of which is organized as a lobby. The organized group receives g_1, the unorganized group $g_2 = g - g_1$. Each group i has a gross utility from government provided goods $V(g_i)$. The lobby make contributions c, so that its net utility is $V(g_i) - c$. The government's objective takes the form

$$G(g_1, c) = \lambda \left[V(g_1) + V(g - g_1) \right] + (1 - \lambda) c \tag{1}$$

where $V(g_1 + V(g - g_1)$ is social welfare.

The politically optimal allocation must satisfy

$$Max \quad V(g_1) - c \tag{2}$$

subject to $G(g_1, c) \geq \widehat{G}$, where $\widehat{G}$ is the maximum of $V(g_1) + V(g - g_1)$. Show that this is equivalent to maximizing $V(g_1) + \lambda V(g - g_1)$.

1.1 Solution

Using (1), the politician must receive a contribution

$$c = \frac{\widehat{G}}{1-\lambda} - \frac{\lambda}{1-\lambda} \left[V(g_1) + V(g - g_1) \right] \tag{3}$$

in order to achieve $\widehat{G}$. Thus, from (2), the optimal allocation maximizes

$$V(g_1) + \frac{\lambda}{1-\lambda} \left[V(g_1) + V(g - g_1) \right] - \frac{\widehat{G}}{1-\lambda} \tag{4}$$

which, for the given $\widehat{G}$, is identical to maximizing $V(g_1) + \lambda V(g - g_1)$.

2 Three Groups, Two Lobbies

Consider the above problem in which there are three groups, two of which are organized as lobbies, say groups 1 and 2. All groups are equal size. Each group i has the same gross utility from government provided goods $V(g_i)$. The lobbies make contributions c_i, implying net utility is $V(g_i) - c_i$. The government allocates a fixed budget g across the three groups, so that $g = g_1 + g_2 + g_3$. Find the optimal solution.

2.1 Solution

The government's objective takes the form

$$G(g_1, g_2, c_1, c_2) = \lambda \left[V(g_1) + V(g_1) + V(g - g_1 - g_2)\right] + (1 - \lambda)(c_1 + c_2) \tag{5}$$

where $V(g_1)+V(g_2)+V(g_3)$ is social welfare. The intuition from the diagram used in class says that we can represent the allocation as that yielding maximum utility for each lobby separately subject to government utility being at least as high as what it would obtain if it did not deal with that lobby, holding the contribution from and the policy for all other lobbies as given.

That is, the politically optimal allocation must satisfy

$$Max \quad V(g_1) - c_1$$

subject to $G(g_1, g_2, c_1, c_2) \geq \widehat{G}_{-1}$, where $\widehat{G}_{-1}$ is maximum government utility if $c_1 = 0$ and c_2 and g_2 are the optimal choices of the other lobbiy. Hence, we can write the maximization problem as

$$\underset{g_1, c_1}{Max} \quad V(g_1) - c_1 + \mu_1 \left(\lambda \left[V(g_1) + V(g_1) + V(g - g_1 - g_2)\right] + (1 - \lambda)(c_1 + c_2)\right) - \widehat{G}_{-1}$$

yielding first-order conditions

$$V'(g_1) + \mu_1 \lambda \left[V'(g_1) - V'(g_3)\right] = 0$$
$$-1 + \mu_1 (1 - \lambda) = 0$$

which implies that

$$V'(g_1) + \frac{\lambda}{1 - \lambda}(V'(g_1) - V'(g_3)) = 0$$

or that

$$V'(g_1) = \lambda V'(g_3) = 0$$

A similar argument shows that at the optimum

$$V'(g_2) = \lambda V'(g_3) = 0$$

which combined with the budget constraint $g = g_1 + g_2 + g_3$ yields the optimal alocation.

Note that the argument in problem 1 can be used to show that the above maximization problem is equivalent to maximizing

$$V(g_1) + V(g_1) + \lambda V(g - g_1 - g_2)$$

which yields the same solution.