Two enzymes immobilized on nonporous polymeric beads. (Problem 3.19 of Shuler & Kargi)
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Two enzymes with the same substrate are co-immobilized on the same surface.

Reaction #1: $S + E_1 \longrightarrow E_1 + P_1$

Reaction #2: $S + E_2 \longrightarrow E_2 + P_2$

Intermediate products P_1 and P_2 combine spontaneously to form the final product P_3.

$P_1 + P_2 \longrightarrow P_3$

Enzyme kinetic constants from the given graph are:

Reaction #1 $v_{m1} := 1.1 \cdot 10^{-5}$ mg/cm²·sec $K_{m1} := 0.025$ mg/cm³ $v_1(s) := \dfrac{v_{m1} \cdot s}{K_{m1} + s}$

Reaction #2 $v_{m2} := 2 \cdot 10^{-5}$ mg/cm²·sec $K_{m2} := 0.11$ mg/cm³ $v_2(s) := \dfrac{v_{m2} \cdot s}{K_{m2} + s}$

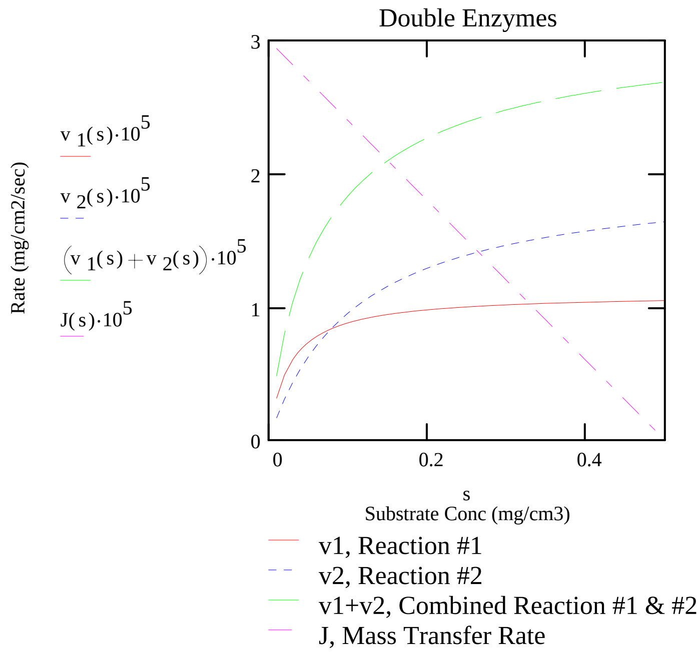
Part a) Find total rate of substrate disappearance, based on the following operating parameters.

Mass transfer coefficient: $k_L := 6 \cdot 10^{-5}$ cm/sec

Substrate concentration in the bulk liquid: $s_b := 0.5$ mg/cm³

Mass transfer $J(s) := k_L \cdot (s_b - s)$

$s := 0.01, 0.02 .. s_b$



When there is only one enzyme present at one time, the intersection of the two curves v_1 & J gives solution to Reaction #1, and that of v_2 and J gives solution to Reaction #2.
When there are two enzymes present simultaneously, the intersection of v_1+v_2 & J give solution to combined Reaction #1 and Reaction #2.

Determine surface concentration of substrate at steady-state:

$s := s_b$ ... Initial guess   Given $J(s) = v_1(s) + v_2(s)$ $s := \mathrm{Find}(s)$ $s = 0.15$ mg/cm^3

Rate of consumption of substrate due to Reaction #1 $v_1(s) = 9.431 \cdot 10^{-6}$ mg/cm^2·sec

Rate of consumption of substrate due to Reaction #2 $v_2(s) = 1.155 \cdot 10^{-5}$ mg/cm^2·sec

Total rate of consumption of substrate due to both reactions $v_1(s) + v_2(s) = 2.098 \cdot 10^{-5}$ mg/cm^2·sec

Mass transfer of substrate to surface (check) $J(s) = 2.098 \cdot 10^{-5}$ mg/cm^2·sec

Part b) Overall effectiveness factor is the ratio of observed rate with mass transfer to the intrinsic rate without mass transfer limitation.

effectiveness factor $\eta := \dfrac{v_1(s) + v_2(s)}{v_1(s_b) + v_2(s_b)}$ $\eta = 0.781$

Part c) Ratio of P_2 to P_1 $\text{ratio} := \dfrac{v_2(s)}{v_1(s)}$ ratio $= 1.225$

Part d) Find s_b that leads to equimolar amount of P_1 and P_2 (i.e., $v_1 = v_2$), while k_L remains unchanged. We first find the value of substrate concentration on the surface such that $v_1 = v_2$.

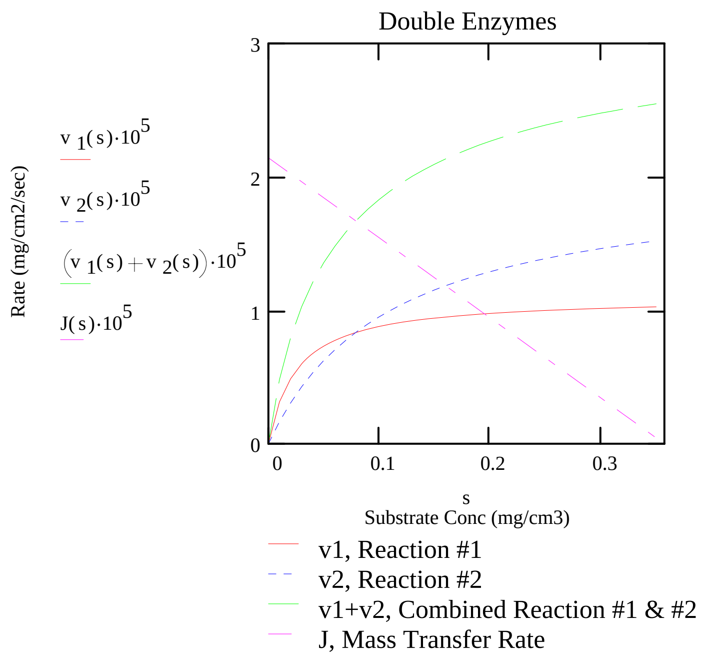
$s := s_b$ ... initial guess   Given $v_1(s) = v_2(s)$ $s := \mathrm{Find}(s)$ $s = 0.079$ mg/cm^3

We then find the value of s_b that satisfies the condition where total rate of substrate consumption equals to rate of substrate mass transfer (i.e., $v_1 + v_2 = J$).

$J(s_b) := k_L \cdot (s_b - s)$

$s_b := s$ ... initial guess   Given $v_1(s) + v_2(s) = J(s_b)$ $s_b := \mathrm{Find}(s_b)$ $s_b = 0.357$ mg/cm^3

Plot $s := 0, 0.01 .. s_b$ $J(s) := k_L \cdot (s_b - s)$



The two curves v_1 & v_2 intersect at the same value of s as the two curves $v_1 + v_2$ & J does.