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THE NONDIMENSIONALIZATION OF EQUATIONS DESCRIBING FLUIDIZATION WITH APPLICATION TO THE CORRELATION OF JET PENETRATION HEIGHT

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Abstract—A nondimensionalization of the basic continuum equations of fluidization and the boundary conditions for these equations provides dimensionless hydrodynamic and geometric parameters which characterize the state of fluidization. For an incompressible gas and monosize, rigid spherical particles in a nonreactive, isothermal environment with no interparticle forces, it is shown that there are four dimensionless hydrodynamic parameters, involving seven physical variables: $\rho^s vd^2/\mu x$, Stokes number; ρ/ρ^s, ratio of gas to solid density; v^2/gx, Froude number; and $\rho vd/\mu$, Reynolds number. The seven physical variables are: ρ (ρ^s), gas (solid) density, v, jet velocity, d, particle diameter, x, bed dimension, g, gravity and μ, gas viscosity. The nature and number of the dimensionless geometric parameters depend upon the geometry of the fluidized bed. These dimensionless parameters are applied to correlating data on jet penetration height, in fluidized beds. For the data on single jets, a correlation of jet penetration height, l, can be represented by:

$$\frac{l}{d_0} = 26.9\left(\frac{v^2}{gd_0}\right)^{0.322}\left(\frac{\rho}{\rho^s}\right)^{0.325}\left(\frac{\rho^s vd^2}{\mu d_0}\right)^{-0.124}$$

where d_0 is the injector diameter. For the data on jets in multiple jet configurations, it is represented by:

$$\frac{l}{d_0} = 55.6\left(\frac{v^2}{gd_0}\right)^{0.251}\left(\frac{\rho}{\rho^s}\right)^{0.322}\left(\frac{\rho^s vd^2}{\mu d_0}\right)^{-0.134}.$$

These correlations are developed from 260 data points for single jets and 122 data points for multiple jets. There is not a Reynolds number, $\rho vd/\mu$, in either of these correlations; for the range of experimental environments in the data, this parameter had little influence on the correlations.

INTRODUCTION

The formation of jets and bubbles at the grid of a fluidized bed have an important influence on both the local and global hydrodynamic mixing in the bed.

Most of the data on jet penetration and bubble formation has been obtained from cold flow experiments. Such experiments were initially done in transparent two-dimensional planar beds, e.g. Wen *et al.* (1977). However, three-dimensional beds with a transparent wall [e.g. Markhevka *et al.* (1971)], or semicircular beds with a flat transparent wall [Yang and Keairns, 1978. 1979; Yang *et al.*, 1984) have been most effectively used in examining jet penetration. The observations of jets in such beds are usually recorded with cine photography. Studies in three-dimensional beds incorporate indirect measurements such as, the variation of density, or changes in momentum with height; the ideal method is X-rays coupled with cine photography, but the power requirements are often prohibitive (Rowe and Everett, 1972).

For the present study, the terminology "jet penetration" is used in a qualitative sense. When a sufficiently high rate of gas flow is introduced through an injector in a gas fluidized bed, transient, devoid of solid particles, forms above the injector and eventually detaches from it. The transient nature of this void is like that of a bubble and it may be appropriate to view the "jet" as a "bubble" [cf. Rowe *et al.* (1979)]. However, the void is often distorted by the momentum of the gas flow [cf. Knowlton and Hirsan (1980) and Hirsan *et al.* (1980)] and a permanent flare is sometimes observed. Rowe *et al.* (1979) studied the formation of bubbles at a single orifice in both incipiently fluidized and completely defluidized beds using X-rays and cine photography. They have suggested a distinction between bubbles and jets based on temporal characteristics rather than shape; only a permanent flare is defined as a jet, whereas the intermittent discharge of a void, regardless of the shape, is understood to be a bubble.

Some progress has been achieved in the correlation of data on jetting phenomena. In particular, there has been success in the correlation of jet penetration height [e.g. Blake *et al.* (1984), Yang *et al.* (1984), Yang (1981), Yang and Keairns (1979, 1978), Wen *et al.* (1977) and Merry (1975)]. A discussion of such work is presented by Yang *et al.* (1984), Blake *et al.* (1984) and Massimilla (1985); Massimilla also reviews other phenomena of gas jets in fluidized beds.

In most of the correlations of jet penetration height, only a limited range of data is used. Blake *et al.* (1984) demonstrated a successful correlation for a rather wide range of data; but the correlation exhibited some divergence between the data of Yang *et al.* (1984) for a large scale rig and that for small scale rigs.

For the present study the data correlated by Blake *et al.* (1984) is revisited and correlated using dimensionless parameters from the continuum theories of fluidization [cf. Murray (1965), Anderson and Jackson (1967) and Sunderland (1986)]. Specifically, continuum equations are nondimensionalized and the dimensionless hydrodynamic and geometric parameters so developed are used to correlate data on jet penetration height. This utilization of the continuum equations is in accord with that of Scharff *et al.* (1978), Fitzgerald and Glicksman (1983) and Glicksman (1984). However, in the present study some differences occur in the choice of reference values for the nondimensionalization and it is claimed that the Stokes number is a most important hydrodynamic parameter for representing fluidization.

BASIC EQUATIONS OF FLUIDIZATION AND NONDIMENSIONALIZATION

The present analysis begins with continuum equations for the solid particle and gas phases in a fluidized bed, such as those developed by Murray (1965) or Anderson and Jackson (1967). However it is also assumed that the fluidization is isothermal, the particles are rigid and spherical, the gas is incompressible and the interparticle forces can be neglected.

We introduce the following reference quantities:

ε, bed voidage
v, gas velocity
x, dimension of bed
ρ^s, solid (grain) density
$\mu x v/d^2$, hydrostatic pressure (note that another choice for this reference pressure could be ρv^2; however, it would not change the nondimensional hydrodynamic parameters. Further, the indicated reference pressure is the appropriate one.

With these reference quantities the equations of Murray (1965) or Anderson and Jackson (1967) can be expressed in dimensionless form. The nondimensional variables are denoted by upper case symbols while the above reference quantities are indicated by lower case symbols.

Conservation of gas mass is:

$$\frac{\partial}{\partial T}\nabla + \frac{\partial}{\partial X_i}(\nabla V_i) = 0 \tag{1}$$

and conservation of gas momentum is:

$$\frac{\rho}{\rho^s}\left\{\frac{\partial}{\partial T}(\nabla V_i) + \frac{\partial}{\partial X_j}(\nabla V_i V_j)\right\} = -\frac{\mu x}{\rho^s v d^2}\left[\nabla\frac{\partial P}{\partial X_i}\right.$$
$$+(1-\varepsilon\nabla)\left\{1+F\left(\frac{\rho v d}{\mu}|V_i - U_i|\right)\right\}\frac{1}{\varepsilon}$$
$$\left.\times G(\nabla,\varepsilon)(V_i - U_i)\right] - \frac{\rho}{\rho^s}\frac{gx}{v^2}\nabla\delta_{i2}. \tag{2}$$

Conservation of solid phase mass is:

$$\frac{\partial}{\partial T}(1-\varepsilon\nabla) + \frac{\partial}{\partial X_i}[(1-\varepsilon\nabla)U_i] = 0 \tag{3}$$

and conservation of solid phase momentum is:

$$\frac{\partial}{\partial T}[(1-\varepsilon\nabla)U_i] + \frac{\partial}{\partial x_j}[(1-\varepsilon\nabla)U_i U_j]$$
$$= \frac{\mu x}{\rho^s v d^2}\left[-(1-\varepsilon\nabla)\frac{\partial P}{\partial X_i}\right.$$
$$+(1-\varepsilon\nabla)\left\{1+F\left(\frac{\rho v d}{\mu}|V_i - U_i|\right)\right\}$$
$$\left.G(\nabla,\varepsilon)(V_i - U_i)\right] - \frac{gx}{v^2}(1-\varepsilon\nabla)\delta_{i2}. \tag{4}$$

In these equations ∇ is the local voidage, V_i (U_i) is the local gas (solid) velocity vector, and P is the local pressure. The functions F and G in the drag coefficient depend on the indicated arguments, and if, for example, the formulation of Wen and Galli (1971) is used: $F\left(\frac{\rho v d}{\mu}|V_i - U_i|\right) = 0.15 \left(\frac{\rho v d}{\mu}\right)^{0.687} |V_i - U_i|^{0.687}$, and $G(\nabla, \varepsilon) = \frac{36}{(\varepsilon\nabla)^{4.7}}$.

The Buckingham π theorem (Shames, 1982) demonstrates that, exclusive of the voidage ε, there should be four dimensionless hydrodynamic parameters, involving seven physical variables, which describe the state of fluidization. While the choice of the four dimensionless hydrodynamic parameters is somewhat arbitrary, the eqs (1)–(4) provide guidance for that choice. First, the convservation of solid phase momentum should domianate fluidized bed mixing, and it is natural that the following dimensionless parameters from eq. (4) be used:

$\frac{\rho^s v d^2}{\mu x}$, ratio of particle inertia to viscous drag force on the particle, where the variable, x, is a dimension of the bed (Stokes number).
$\frac{v^2}{gx}$, ratio of inertia to gravity force (Froude number) and
$\frac{\rho v d}{\mu}$, ratio of fluid inertia to viscous drag force on the particle (Reynolds number based on the particle diameter).

With these three dimensionless parameters, and with:

$\frac{\rho}{\rho^s}$, ratio of gas density to solid density,

all of the dimensionless parameters in the conservation of gas momentum, eq. (2), are obtained.

The first of these dimensionless parameters is new to the analysis of fluidization and it is fundamental to the present approach. However, parameters similar to $\rho^s v d^2/\mu x$ occur in several analyses of gas and particle flows. For example, Marble (1970) has studied lightly loaded gas–particle flows with small particles; he introduced the parameter $\rho^s v d^2/18\mu x$, measuring the gas–particle equilibration time relative to the time, x/v, which characterizes the gas motion. For the small particles considered by Marble (1970), $\rho^s v d^2/\mu x$ is very small compared with unity, while in the fluidized environments considered herein, $\rho^s v d^2/\mu x$ is very large compared with unity. Again, it is possible to combine these four dimensionless hydrodynamic parameters and obtain other dimensionless parameters. Therefore, the choice of these parameters is not unique; however, it seems preferable to use these four parameters which are obtained from the basic equations. For limiting behavior of the basic equations, as in the case of minimum fluidization, it is natural that some of these dimensionless parameters will be combined.

Also, it appears that in other limiting cases, the present formulation facilitates the simplification of the nondimensionalization. For example, if gas inertia and particle Reynolds number are small only $\rho^s v d^2/\mu x$ and v^2/gx are needed to characterize fluidization. Such conclusions are summarized in Table 1

The above analysis of the equations utilizes a single geometric length scale for the bed geometry. Additional geometric length scales will occur if one accounts for the boundary conditions (or the initial conditions) of those equations. Since these boundary conditions would be imposed at the geometric boundaries of the bed, the physical locale of such boundary conditions would appear, in dimensionless format, as a ratio to that bed dimension, x. Therefore, the complete statement of most boundary value problems in the theory of fluidization will involve multiple length scales. Also, the length scales of hydrodynamic phenomena, such as bubble diameter or jet height would appear, in dimensionless format, as ratios to the bed dimension, x. And, naturally, if the theory is a valid guide to the experimental realm, then such length scales should occur in the correlations.

It is noted that the four dimensionless hydrodynamic parameters in this study are different from those obtained by Glicksman (1984, 1988). There is a certain arbitrariness in selecting such parameters, and the relative merits of say, the Stokes number *vis-à-vis* other dimensionless parameters used by Glicksman (1984) must be assessed for each experimental environment. It is remarked that the present formulation permits the use of fewer dimensionless parameters in some cases. Again, when fluid inertia and particle Reynolds number are sufficiently small, the present analysis indicates that two hydrodynamic parameters characterize fluidization. That corresponds to Glicksman's (1984) characterization, through three parameters, of the "viscous limit". In Glicksman (1988) that viscous limit is reexamined and a two parameter characterization, involving a Froude number and modified Reynolds number, is presented.

An important physical implication in the present study [and that of Sunderland (1986)] is that, for the entire range of fluidization, the ratio of particle diameter to bed dimension, d/x, does not, by itself, occur as a dimensionless parameter. Hence d/x, would not, by itself, affect any scaling of experimental data. That ratio would only affect such scaling through the weighted influence of the other physical variables in the Stokes number. Again, the Stokes number measures the "stopping distance" of the particle relative to the bed dimension and this stopping distance rather than the particle size affects the hydrodynamics. For the special case of the "viscous limit" (small fluid inertia and small particle Reynolds number) Glicksman (1988) presents an analogous conclusion.

Table 1. Dimensionless hydrodynamic parameters for isothermal fluidization with incompressible fluid and rigid spherical monosize particles

State of fluidization	Nondimensional parameters
General	$\frac{\rho v d}{\mu}, \frac{\rho^s v d^2}{\mu x}, \frac{v^2}{gx}, \frac{\rho}{\rho^s}$
Fluid inertia negligible relative to solid inertia	$\frac{\rho v d}{\mu}, \frac{\rho^s v d^2}{\mu x}, \frac{v^2}{gx}$
Fluid inertia negligible relative to solid inertia, and low particle Reynolds number	$\frac{\rho^s v d^2}{\mu x}, \frac{v^2}{gx}$

CORRELATION OF JET PENETRATION HEIGHT MEASUREMENTS

In the present review of the experiments of the many investigators, the published data have been correlated through the use of a linear regression analysis. A summary of the experiments is provided in Table 2, where the ranges of the dimensionless hydrodynamic parameters are displayed. In Ku (1982), the data were obtained in a temperature range between 20°C and 700°C. The measurements of Knowlton and Hirsan (1980) were made at a room temperature but in a pressure range between 440 and 5265 kPa (50–750 psig). Also there are differences in geometry between the many experimental environments. Semicircular beds, including a single jet, are used in Ku (1982), Yang and Keairns (1978), Knowlton and Hirsan (1980), Yang *et al.* (1984) and Sit (1981), while three-dimensional configurations with a single jet (Markhevka *et al.*, 1971) or with multiple jets (Deole, 1980; Behie *et al.*, 1971; Tanaka *et al.*, 1980; Basov *et al.* 1969) are used in the remaining studies.

Each investigator has a specific definition of jet penetration height; such definitions are followed in this study. In the cases where maximum and minimum values are reported (Yang and Keairns, 1978; Knowlton and Hirsan, 1980), a mean value is used.

Single jet data is available from Markhevka *et al.* (1971), Ku (1982), Yang and Keairns (1978), Knowlton and Hirsan (1980), Sit (1981) and Yang *et al.* (1984); there are a total of 260 data points in these references.

Table 2. Summary of data and numerical values of dimensionless hydrodynamic and geometric parameters for correlating jet penetration height

Group investigator	$\dfrac{\rho^s v d^2}{\mu d_0}$	$\dfrac{v^2}{g d_0}$	$\dfrac{\rho v d}{\mu}$	$\dfrac{\rho}{\rho^s}$	$\dfrac{d_0}{d}$	$\dfrac{d_0}{d_b}$
Ku (1982) □	0.903×10^4–0.142×10^8	0.61×10^3–0.40×10^7	0.99×10^2–0.12×10^5	0.46×10^{-3}	0.18×10–0.24×10^2	0.55×10^{-2}–0.24×10^{-1}
Ku (1982) ○	0.846×10^4–0.168×10^8	0.17×10^4–0.27×10^8	0.34×10^2–0.42×10^4	0.14×10^{-3}–0.2×10^{-3}	0.18×10–0.24×10^2	0.55×10^{-2}–0.24×10^{-1}
Doele (1980) ▲	0.103×10^5–0.241×10^7	0.26×10^4–0.15×10^7	0.11×10^3–0.71×10^4	0.46×10^{-3}	0.64×10–0.24×10^2	0.12×10^{-1}
Behie *et al.* (1971) ■	0.203×10^3–0.613×10^3	0.42×10^4–0.25×10^5	0.92×10^2–0.18×10^3	0.12×10^{-2}	0.25×10^3–0.38×10^3	0.45×10^{-1}–0.68×10^{-1}
Markheva *et al.* (1971) *	0.130×10^4–0.191×10^5	0.12×10^4–0.25×10^6	0.89×10^2–0.13×10^4	0.12×10^{-2}	0.57×10^2	0.27×10^{-1}
Basov *et al.* (1969) ●	0.404×10^3–0.183×10^5	0.26×10^3–0.14×10^6	0.37×10^2–0.55×10^3	0.12×10^{-2}	0.24×10^2–0.16×10^3	0.6×10^{-2}–0.4×10^{-1}
Tanaka *et al.* (1980) ◆	0.294×10^4–0.176×10^6	0.19×10^3–0.27×10^6	0.36×10^2–0.87×10^3	0.45×10^{-3}	0.11×10^2–0.33×10^2	0.2×10^{-1}–0.6×10^{-1}
Knowlton and Hirsan (1980) ×	0.351×10^3–0.790×10^5	0.41–0.11×10^5	0.47×10^3–0.11×10^5	0.12×10^{-2}–0.23×10^{-1}	0.58×10^2–0.61×10^2	0.83×10^{-1}
Yang and Keairns (1978) △	0.715×10^4–0.775×10^5	0.28×10^2–0.28×10^4	0.6×10^3–0.6×10^4	0.57×10^{-2}	0.14×10^2–0.19×10^2	0.22–0.32
Yang *et al.* (1984) ⊞	0.859×10^3–0.486×10^4	0.21×10^2–0.60×10^2	0.48×10^3–0.14×10^4	0.11×10^{-2}	0.26×10^3–0.61×10^3	0.13
Sit (1981) ▽	0.177×10^5–0.426×10^5	0.77×10^4–0.45×10^5	0.42×10^3–0.10×10^4	0.11×10^{-2}	0.22×10^2	0.43×10^{-1}

Again, these measurements of jet penetration height have been analyzed with a linear regression analysis.

A very good correlation, within $\pm 40\%$, of the jet penetration height, l, is obtained for this single jet data. The correlation is based on the three hydrodynamic parameters, $\rho^s dv^2/\mu d_0$, ρ/ρ^s, v^2/gd_0:

$$\frac{l}{d_0} = 26.9\left(\frac{v^2}{gd_0}\right)^{0.322}\left(\frac{\rho}{\rho^s}\right)^{0.325}\left(\frac{\rho^s v d^2}{\mu d_0}\right)^{-0.124} \tag{5}$$

This correlation is shown in Fig. 1 and the definition of the symbols is indicated, again, in Table 2. Certainly, other combinations of parameters may be attempted to improve or simplify the correlation. Without proof it is noted that the use of only the two hydrodynamic parameters v^2/gd_0, ρ/ρ^s yields significant scatter in the data points, and that the use of the four parameters $\rho v d/\mu$, $\rho^s v d^2/\mu d_0$, v^2/gd_0, ρ/ρ^s does not provide a measurable improvement in the correlation of eq. (5). It is suspected that the lack of influence of $\rho v d/\mu$ on the correlation of jet penetration height is a consequence of the limited range of that parameter in the data base shown in Table 2.

Jet height data for individual jets in multiple jet environments is available from Tanaka *et al.* (1980), Deole (1980), Behie *et al.* (1971) and Basov *et al.* (1969); there are a total of 120 data points on jet penetration height in these references. A good correlation, within $\pm 40\%$, of the jet penetration height is obtained for this data base, with the same three hydrodynamic parameters $\rho^s v d^2/\mu d_0$, ρ/ρ^s, v^2/gd^0 which are applied to the correlation of single jet data in the previous eq. (5). The correlation is:

$$\frac{l}{d_0} = 55.6\left(\frac{v^2}{gd_0}\right)^{0.251}\left(\frac{\rho}{\rho^s}\right)^{0.322}\left(\frac{\rho^s v d^2}{\mu d_0}\right)^{-0.134}. \tag{6}$$

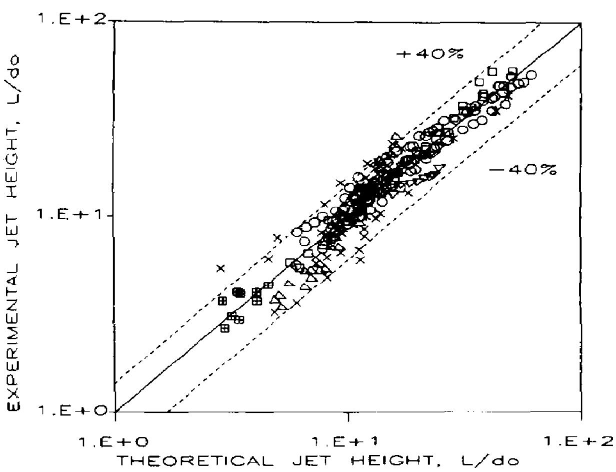

Fig. 1. Preferred correlation, based on three parameters, of the single jet penetration height data, eq. (5):

$$\frac{l}{d_0} = 26.9\left(\frac{v^2}{gd_0}\right)^{0.322}\left(\frac{\rho}{\rho^s}\right)^{0.325}\left(\frac{\rho^s v d^2}{\mu d_0}\right)^{-0.124}.$$

For symbols see Table 2.

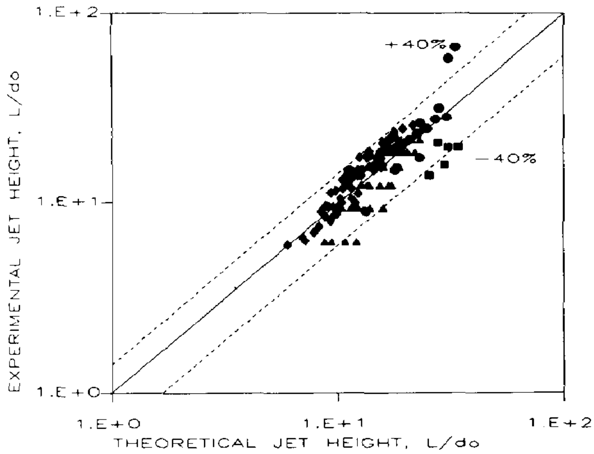

Fig. 2. Preferred correlation, based on three parameters, of the jet penetration height data, in a configuration of multiple jets, eq. (6):

$$\frac{l}{d_0}=55.6\left(\frac{v^2}{gd_0}\right)^{0.251}\left(\frac{\rho}{\rho^s}\right)^{0.322}\left(\frac{\rho^s vd^2}{\mu d_0}\right)^{-0.134}.$$

For symbols see Table 2.

This correlation for jet penetration height is shown in Fig. 2 and the definition of the symbols is indicated in the previous Table 2. The dependence of this correlation, eq. (6), upon power coefficients of the hydrodynamic parameters $\rho^s vd^2/\mu d_0$, ρ/ρ^s, v^2/gd_0 is consistent with that for the single jet in eq. (5). The only significant difference is that eq. (6) has a larger leading coefficient. It is speculated that this leading coefficient reflects a competition between the enhancement of the jet height by adjacent jets and the diminishment of the jet height by the wall. From eqs (5) and (6), the adjacent jets appear to increase the jet height relative to that associated with isolated jets. Unfortunately, the data base is not adequate to validate this speculation. However, there is some evidence to support the concept that geometry is important. For example, in the case of the single jet data, the addition of the geometric parameter d_b/d_0, where d_b is the diameter of the rig, leads to a correlation of the single jet penetration height data as:

$$\frac{l}{d_0}=26.9\left(\frac{v^2}{gd_0}\right)^{0.308}\left(\frac{\rho}{\rho^s}\right)^{0.339}\left(\frac{\rho^s vd^2}{\mu d_0}\right)^{-0.121}\times\left(\frac{d_b}{d_0}\right)^{0.062}. \tag{7}$$

Conversely, for the multiple jet configurations, the inclusion of another geometric parameter d_n/d_o, where d_n is the pitch of the grid injector array, leads to the correlation of the jet height data as:

$$\frac{l}{d_0}=22.1\left(\frac{v^2}{gd_0}\right)^{0.268}\left(\frac{\rho}{\rho^s}\right)^{0.189}\left(\frac{\rho^s vd^2}{\mu d_0}\right)^{-0.163}\times\left(\frac{d_b}{d_0}\right)^{0.077}\left(\frac{d_n}{d_0}\right)^{-0.102}. \tag{8}$$

Equations (7) and (8) suggest the influences of geometry on the coefficients of respectively eqs (5) and (6). It is interesting to note that the addition of the geometric parameters d_b/d_0, d_n/d_0 affects both the lead coefficient and the powers of the hydrodynamic parameters in the correlations for the multiple jet environment [eqs (6) and (8)]. This may be indicative of nonlinear hydrodynamic influences when multiple jets interact. For a jet in a multiple jet environment eq. (8) indicates that an increase in the pitch will decrease the jet height. For both single and multiple jet environments, eqs (7) and (8) show that an increase in rig diameter will increase the jet height. These are intriguing possibilities, but it should be emphasized that the data base is small. Further, the dependence of the respective correlations on these geometric parameters is rather weak. The addition of the geometric parameters in eqs (7) and (8) does not significantly improve the correlations of jet penetration height. Specifically, the addition of d_b/d_0 in eq. (7) provides a slightly better correlation than eq. (5) of the single jet height through the inclusion of additional data points within the $\pm 40\%$ band. However, for the correlation of multiple jet environments there is no measurable improvement with the addition of d_b/d_0 and d_n/d_0 through eq. (8).

Consequently, based on accuracy and simplicity, the preferred correlations for jet penetration in single and multiple jet environments are, respectively, eqs (5) and (6).

Blake *et al.* (1984) present a correlation of jet penetration height data, which is based upon the parameters $\rho vd/\mu$, ρ/ρ^s, v^2/gd_0. In Fig. 3, that correlation by Blake *et al.* of the combined jet penetration height data for jets in single and multiple configurations is shown. It is evident that the data of Yang *et*

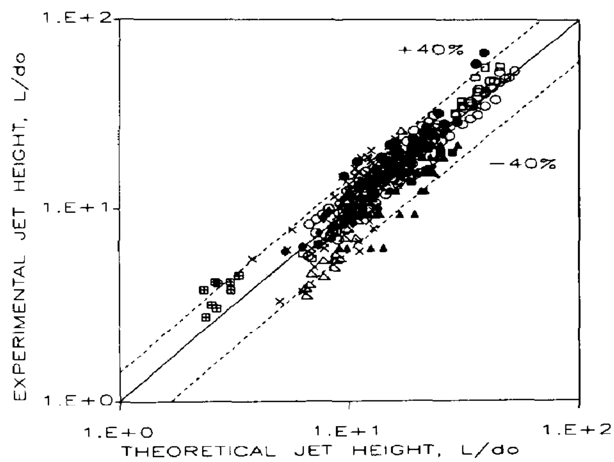

Fig. 3. Correlation of jet penetration height data in single and multiple jet configurations by Blake *et al.* (1984). Particle Reynolds number rather than Stokes number is used in this correlation.

$$\frac{l}{d_0}=110\left(\frac{v^2}{gd_0}\right)^{0.304}\left(\frac{\rho}{\rho^s}\right)^{0.513}\left(\frac{\rho vd}{\mu}\right)^{-0.189}.$$

For symbols see Table 2.

al. (1984), while within the $\pm 40\%$ lines, appear to be isolated from the other data. Also, Blake *et al.* (1984) shows that the Yang and Keairns (1978) data exhibit a slightly different dependence on the hydrodynamic parameters than that of the other data. A correlation of that combined data base using the Stokes number $\rho^s v d^2/\mu d_0$ rather than the Reynolds number $\rho v d/\mu$ is shown in Fig. 4. A comparison between Fig. 3 and 4 shows that the present use of the Stokes number $\rho^s v d^2/\mu d_0$ provides a more uniform representation of the data. Specificially, the Yang *et al.* (1984) and Yang and Keairns (1978) data exhibit a dependence upon the hydrodynamic parameters which is more consistent with the data of the other investigators. This supports the speculation that the Stokes number is a fundamental parameter in representing fluidization.

CONCLUDING REMARKS

The nondimensionalization of the basic continuum equations of fluidization and the boundary conditions for these equations yields dimensionless hydrodynamic and geometric parameters which characterize the state of fluidization. For an incompressible gas and monosize, rigid spherical particles in a nonreactive, isothermal environment with no interparticle forces, it is shown that there are four dimensionless hydrodynamic parameters, involving seven physical variables: $\rho^s v d^2/\mu x$, Stokes number; ρ/ρ^s, ratio of gas to solid density; v^2/gx, Froude number; and $\rho v d/\mu$, Reynolds number. The seven physical variables are: ρ (ρ^s), gas (solid) density, v, gas velocity, d, particle diameter, x, bed dimension, g, gravity and, μ, gas viscosity. The nature and number of the dimensionless geometric parameters depend upon the geometry of the fluidized bed.

Correlations of jet penetration height have been developed, which describe the data for a range of experimental environments. These correlations also show that the Stokes number $\rho^s v d^2/\mu d_0$, where d_0 is the injector diameter, is important in scaling fluidization phenomena.

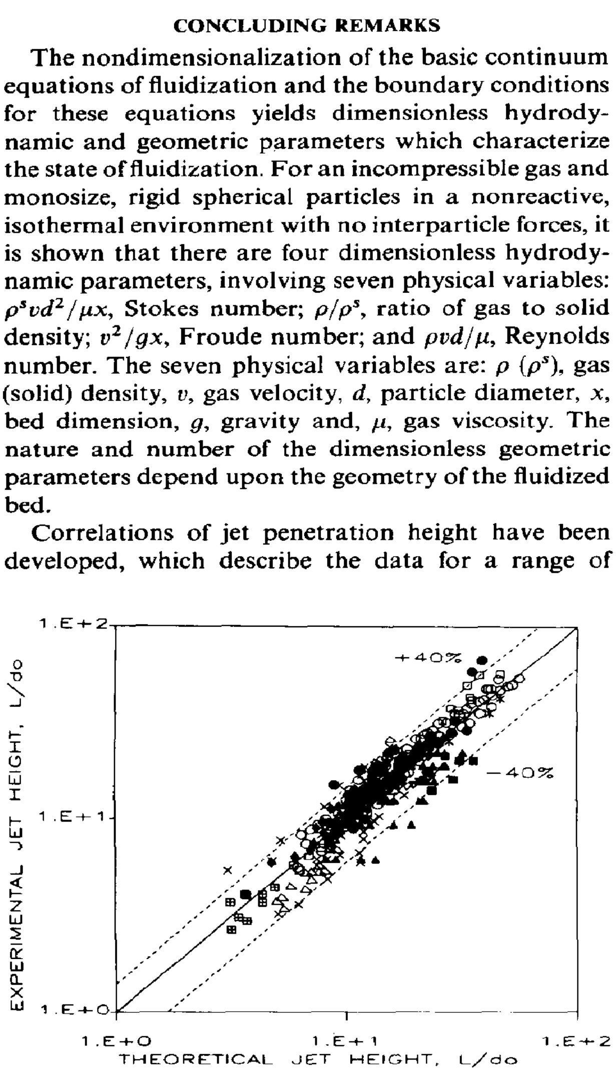

Fig. 4. Correlation of jet penetration height data in single and multiple jet configurations using Stokes number.

$$\frac{l}{d_0} = 27.5\left(\frac{v^2}{gd_0}\right)^{0.312}\left(\frac{\rho}{\rho^s}\right)^{0.304}\left(\frac{\rho^s v d^2}{\mu d_0}\right)^{-0.131}$$

For symbols see Table 2.

The Stokes number is known to be an important parameter in the dynamics of gas–particle flows in the absence of particle–particle forces (Marble, 1970; Hinze, 1972). Indeed, if the equations in the present study were generalized to include a viscous stress tensor for the gas phase, the equations would be formally identical to those for such gas–particle flows. A nondimensionalization would yield the same four hydrodynamic parameters as used herein, together with a fifth parameter $\rho v x/\mu$, a Reynolds number based on the bed dimension. Again, the Buckingham π theorem indicates that only four of these five hydrodynamic parameters are independent and the choice of the four parameters is occasioned by the regime of interest.

It is acknowledged that the dimensionless parameters, in the present study, were obtained from rather restrictive assumptions on the basic equations. Clearly, these simplifications would have to be eliminated in a more general analysis of fluidization. A primary simplification is the neglect of a shape factor, ϕ_s. However, there is a dearth of information on shape factor in the data which are used herein; further it is easy to add ϕ_s to the present analysis. A more difficult issue is the neglect of interparticle forces or solid phase rheology. Such solid phase rheology has been represented as Newtonian by Pritchett *et al.* (1978). However, more complex models are currently proposed [e.g. Syamlal (1987)] which are based on granular flow theories [cf. Lun *et al.* (1984) and Johnson and Jackson (1987)]. If the granular–collisional theory of Lun *et al.* (1984) were used to represent the solid phase rheology, it would provide only one more dimensionless hydrodynamic parameter than the present analysis. Specifically, a coefficient of restitution, which measures the inelastic nature of particle collisions, would be added. If particle–particle friction dominates the granular flow (Johnson and Jackson, 1987) then a solid pressure coefficient and an internal angle(s) of friction, would be added to the dimensionless parameters of the present study.

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NOTATION

d particle diameter, cm
d_0 injector diameter, cm
d_b bed diameter, cm

d_n pitch, cm
g gravitational acceleration, cm/s^2
l jet height, cm
P normalized pressure
T normalized time
U_i normalized solid phase velocity
v reference gas velocity, cm/s
V_i normalized gas velocity
x reference bed dimension, cm
X_i normalized coordinate

Greek letters
ε reference voidage
∇ normalized voidage
μ gas viscosity, g/cm s
ρ gas density, g/cm^3
ρ^s solid density, g/cm^3

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