Liberalism Against Populism

A Confrontation Between the Theory of Democracy and the Theory of Social Choice

William H. Riker
University of Rochester

Prospect Heights, Illinois

For information about this book, write or call:

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The Connection Between the Theory of Social Choice and the Theory of Democracy

The theory of social choice is a theory about the way the tastes, preferences, or values of individual persons are amalgamated and summarized into the choice of a collective group or society. Because voting is one method of aggregating values, social choice theory must include, among other elements, a theory of voting. Voting is in turn an indispensable feature of democracy because, however the goals of democracy are defined, its method involves some kind of popular participation in government. Although participation can take many forms, historically—and probably logically—it invariably includes voting. Therefore, the theory of social choice is highly relevant to the theory of democracy.

Only recently, however, has this relevance been recognized. One reason recognition has come slowly is that students of democracy have tended to regard the mechanism of voting and counting votes as a trivial subject. There has, it is true, been a century-long controversy over proportional representation, but that controversy has centered more on fairness than on the operating characteristics of alternative mechanisms. Otherwise, political philosophers, engaged in the pursuit of justice, have ignored and neglected the theory of voting methods as something best left to the attention of municipal employees.

To some degree, the philosophers have been right: The theory of voting has barely existed until this generation, and one cannot attribute much relevance to a theory that barely exists. Only in the 1940s did Duncan Black, then an unknown lecturer on economics in Glasgow, rediscover the paradox of voting (see section 1.H) and recognize the full significance of a theory of social choice.

The paradox of voting is the coexistence of coherent individual valuations and a collectively incoherent choice by majority rule. In an election with three or more alternatives (candidates, motions, etc.) and three or

more voters, it may happen that when the alternatives are placed against each other in a series of paired comparisons, no alternative emerges victorious over each of the others: Voting fails to produce a clear-cut winner. This paradox was originally discovered by Condorcet in the late eighteenth century, just at the time that voting was becoming a much more frequently used method of social decision. Condorcet's discovery made little impression on his contemporaries, and neither did the rediscoveries in the late nineteenth and early twentieth centuries by Lewis Carroll (Charles Dodgson) and E. J. Nanson make much impression on their contemporaries. Duncan Black, however, in a series of essays begun in the 1940s and culminating in his *Theory of Committees and Elections,* effectively communicated the profound significance of the rediscovered paradox to other scholars.[1] One was Kenneth Arrow, who proved in *Social Choice and Individual Values* that the paradox may occur in any reasonably fair system of counting votes.[2] Another was Robin Farquharson, who showed in *Theory of Voting* that political strategy and dissimulation about tastes were ineradicable parts of the process of voting.[3]

The work of Black, Arrow, and Farquharson launched the theory of social choice and connected it logically with the theory of democracy. But, though the connection is now made, it is on the whole still true that political scientists and political philosophers have not worked out the significance of the connection. Most writers simply ignore the problem, and those who recognize it tend to sweep it under the rug.[4] Robert Dahl did try to work out some connection between the two theories.[5] But social choice theory, at the time he wrote, was not yet mature. Consequently, the full significance of the connection was not visible to him.

Now, however, that significance can be specified more easily: Democracy is an ideal of both justice and political life, and it is a method of realizing that ideal in ordinary politics. The ideal is individual self-realization (that is, the achievement of the human potential for good qualities of character and behavior) and individual self-respect (that is, a sense of one's worthiness as a person and a pride in one's self-realization). The method is, for each person, free and equal participation in the political life of the community, engaging thus in whatever control of the social environment is possible. Both parts, the ideal and the method, are necessary for democracy. They can exist separately in other contexts—the ideal in, for example, an ethical system and the method in, say, a religious society. But what makes democracy unique is that the democratic means and the democratic ends are joined. Indeed, they are the same things viewed ideally and instrumentally. According to democratic theory, democratic ends can be achieved by democratic means. Now, of course, that assertion may or may not be true. If it is true, then the notion of democ-

racy makes sense. But if it is not true, if the method cannot realize the ideal, then, however lofty the ideal may be, the notion of democracy is meaningless. Thus, a profoundly important question about democracy is whether the means are capable of achieving the ends.

The theory of social choice permits us to approach, and perhaps even partially to answer, that question. The ideal of democracy is set forth in a normative statement of what we want the natural world of human interaction to yield for us. The theory of social choice is an analytic theory about the way that natural world can work and what kinds of outputs that world can yield. By means of this analytic theory, we can discover whether pursuit of the ideal is promising or futile.

What we hope to have is always conditional on what we can have. To seek what we know, a priori, we cannot get is about like trying to square the circle. But to search for an algorithm to compute a result that we know is, at least in principle, computable is a sensible task. This then is the connection between the theory of democracy and the theory of social choice: By use of the latter it is possible to assess, at least in part, whether it is sensible to pursue democratic ends by democratic means.

1.A. The Attainability of Justice

In the study of justice the need to investigate whether the specified means can be used to achieve the specified ends recurs frequently. It is exactly such investigation that has heretofore been neglected by students of democracy.

Such neglect is common enough in contemporary political thought, and many recent interpreters of nondemocratic notions of justice have been equally guilty of neglecting the study of institutions by which their versions of justice might be attained. Witness, for example, John Rawls' utilitarianism with a Kantian overlay or Herbert Marcuse's improbable combination of humanism and violence. Both of them have been presented as definitions of goals, but they lack any consideration at all of whether the goals are physically attainable by any imaginable means.

It does not really make sense to ignore the question of attainability, and in the long history of the study of justice such questions have typically been a central part of the inquiry. In the *Republic,* the earliest-recorded well-articulated theory of justice, Plato defines justice as a condition in which everyone is doing the job best fitted to his or her talents. This definition is only the beginning of a discussion of whether that particular goal can be achieved by various devices, such as an appropriate ideology

(the so-called noble lie), an educational system for rulers, the generation of wisdom-loving kings, and even divine intervention in history. Similarly, in the modern world, the Marxian definition of justice as the distribution of material goods according to needs was proposed along with a concrete method of achieving it—namely, the dictatorship of the proletariat led by its vanguard, the Communist party. Marx himself devoted much thought to concrete steps in the revolution (for example, in the *Critique of the Gotha Program*), and his successors Lenin and Mao equated philosophy with a theory of party organization and a theory of propaganda.

In the interpretation of both the Platonic and the Marxian theories, it is certainly appropriate and in fact customary to inquire whether the means are efficient for attaining the ends. Very few people have seriously considered attempting to achieve Platonic justice, not primarily because Platonic justice is rather uninspiring, but because the education and the institution of a philosopher-king seem internally contradictory and quite unlikely to achieve the intended effects even if they could be carried through. Similarly, although the materialism of Marxist goals has a wide appeal, it is far from clear that the dictatorship of the Communist party has promoted distribution according to need. Many observers believe that Communist bureaucracies, multiplying disincentives for production and satiating themselves with perquisites, have produced distributions even less in accord with stated Marxist goals than was achieved under the systems they replaced. If so, then Marxist means may well have *prevented* the achievement of Marxist ends.

As these two examples indicate, the question of whether particular goals are achievable by specified means (or indeed by any means at all) is an elementary and unavoidable question about *any* theory of justice. Yet up to this time it has not been asked about democracy. The main reason it has not been asked is, I believe, that we have lacked an appropriate base for questioning. But now that we have an analytic theory about the main institution of democracy—namely, voting—we do have an appropriate base. Consequently, it seems both possible and worthwhile to study the relation of democratic means to democratic ends, and that is the purpose of this book.

1.B. The Elements of Democracy

To begin the investigation we need some agreed-upon notion of what democracy is. Unfortunately we cannot go to a unique authoritative source for a definition. Democracies have existed in the ancient Mediter-

ranean world, in late-medieval central European cities, and in many nations of the modern world. Their social and cultural circumstances have been extremely diverse, and their goals and methods have been defined in many constitutions and in a vast body of judicial, philosophical, didactic, and popular commentary. Since the literature celebrating democracy—even authoritatively explicating it—is far too much for one person to read, it is difficult to set forth a fair and inclusive definition.

On another occasion I attempted to by-pass this difficulty by the statistician's device of selecting a sample. I compared five representative documents, looking for the elements they had in common.[6] Without repeating that analysis here, I will list the properties found in those documents. They are participation, liberty, and equality. Most recent writers attribute those properties to democracy; I will explain how they fit together coherently.

To anticipate my conclusion, and to indicate the direction of the argument, I want to point out that the coherence depends on the fact that all democratic ideas are focused on the mechanism of voting. All the elements of the democratic method are means to render voting practically effective and politically significant, and all the elements of the democratic ideal are moral extensions and elaborations of the features of the method that make voting work. Voting, therefore, is the central act of democracy, and this fact makes apparent the immediate relevance of the theory of social choice.

Participation

The crucial attribute of democracy is popular participation in government. This is what the root of the word originally meant in Greek. Although the institutions of participation have been many and varied, they have always revolved around the simple act of voting. Even recent theories, such as those from Dahl and his followers, that equate democracy with the free interplay of groups and the existence of an opposition cannot avoid an emphasis on voting as the ultimate way groups and oppositions make themselves felt. Voting, however, is not equivalent to democracy. Only voting that facilitates popular choice is democratic. This condition excludes voting both in oligarchic bodies and in plebiscites in communist and military tyrannies, where voting is no more than forced approbation. Thus one can say that democracy implies voting but voting does not imply democracy. Or, voting is a necessary, but not sufficient, condition of democracy. To render them equivalent, voting must be surrounded with numerous institutions like political parties and free speech, which organize voting into genuine choice.

The purpose of participation is twofold. In most cases it has been instituted to restrain oppressive rule by subjecting rulers to popular judgment. But, in addition, it has been invested with a positive value of its own. Ruling and being ruled in turn is, said Aristotle, the essence of good citizenship; and good citizenship he equated with the good life. To take, thereby, full responsibility for oneself—both by internal discipline and by cooperative management of the physical and social environment—is to achieve as much self-control as one can. And self-control is a necessary instrument of that human dignity and self-respect that moral philosophers of almost all persuasions have regarded as the best human achievement. To facilitate that achievement is the goal of democracy and its ideal of justice. Hence, participation is an end in itself as well as a practical method.

Liberty

A second feature of democracy is liberty to pursue one's goals. This notion has been variously expressed. In the tradition of Locke, which has dominated Anglo-American thought, liberty has been described as the natural rights inherent in human life and independent even of citizenship. Many of the great democratic declarations are tabulations of rights such as free speech, religious liberty, fair legal procedure, property ownership, and economic security. But the connection between democratic liberty and natural rights is not necessary. In the ancient world there was no notion of natural rights, yet Pericles praised freedom as one of the main features of Athenian democracy. And in the tradition of Rousseau, liberty resides in participation in government, not in rights distinct from government.

Nevertheless, however expressed, there is a close connection between liberty and democracy. How can the persistence of this association be explained? Historically, at least, the association is instrumental because liberty is necessary to organize participation in government. In the English tradition, limited government originated in claims of freedoms against the Crown. The earliest such claim was that members of Parliament not be prosecuted for speeches in Parliament. The claim protected politicians temporarily in office and not certain of staying there and thereby enabled them to form factions and organize voting against the government. Ultimately this freedom was extended to everybody, but it has never lost its association with political opposition and the nourishment of faction. Almost everything else that we think of as civil liberties (the rights of a speedy trial, habeas corpus, and security against unreasonable search and seizure, for example) originated to protect politicians who

feared prosecution if and when they lost office. Thus the historic purpose of these fundamental democratic liberties has been not to provide freedom as an end in itself, but to render effective both political participation and the process of choice in voting.

Freedom, however, has also become an end in itself because, like participation, it generates self-control and facilitates self-respect. Hence it is not only an instrument for, but also a part of, justice. Civil liberties are now thought to be good for everybody, not just politicians. Religious liberty, which religious factions—uncertain of victory in the wars of religion—devised to protect themselves, is now thought to be part of self-control and the good life itself. Economic liberty—that is, the free markets and free entrepreneurship of modern capitalism—originally protected a faction, namely, merchants, in conflict with the Crown and the feudal order. Although it has been fashionable in this century to deride economic freedom, capitalism remains essential for faction: No government that has eliminated economic freedom has been able to attain or keep democracy, probably because, when all economic life is absorbed into government, there is no conceivable financial base for opposition. But economic liberty is also an end in itself because capitalism is the driving force for the increased efficiency and technological innovation that has produced in two centuries both a vast increase in the wealth of capitalist nations and a doubling of the average life span of their citizens. These practical achievements also facilitate self-control and are therefore important features of democratic justice.

Altogether, therefore, democratic liberty (whether civil, religious, or economic) originates as an instrument to organize voting and popular participation in government. Once in existence, however, it has always been found good in itself as a part of self-control and human dignity. So, like participation, democratic liberty—originally an instrument—became a part of the democratic ideal.

Equality

A third feature of democracy is equality, which like liberty and participation, originated in some rough sense as an instrument of voting. Voting would not mean much if each person's vote were not counted in the same way. So equality at the ballot box, by some measure, is necessary to make voting and participation work.

But the claim of equality usually involves much more. Sometimes it means equality before the law, which prevents powerful persons from using the law to take advantage of weaker persons. Sometimes it means equal educational or economic opportunity or even equal shares of the

wealth of the world. Whatever form the claim takes, its moral significance is clear: To permit serious inequality means to deny to some people the chance to the self-control and cooperative management involved in democratic justice. Equality thus becomes an instrument facilitating self-respect and self-realization, although like the other elements of the democratic ideal its logical base lies in the instrumental value of making voting work.

1.C. The Meaning of Democracy

Democracy is both an ideal and a method. Now, having catalogued its features, I can explain how the ideal and the method are assumed to cohere, though, as will be seen, there may be profound philosophical difficulties beneath this assumed coherence. The ultimate moral ideal of democracy is the self-respect and self-realization that are made possible by self-control and the concomitant cooperative control of the environment. Whether that ideal is achieved depends on how individuals view themselves and what they themselves do to realize their potentials. The function of political justice is to facilitate that achievement by creating appropriate social conditions. In a society characterized by democratic justice, people are free (by reason of democratic liberty) and have the chance (by reason of democratic equality) to seek self-respect and self-control (through some kind of democratic participation). The democratic method that is supposed to achieve this ideal is, of course, the same three features viewed as means rather than as ends: The method is the process of participation, specifically through voting, in the management of society, where voting is understood to include all the ancillary institutions (like parties and pressure groups) and social principles (like freedom and equality) that are necessary to render it significant.

Consequently, we can say that voting, which is a main subject in the theory of social choice, is at the heart of both the method and the ideal of democracy. Clearly, therefore, the theory of democracy must be intimately involved with the theory of social choice.

1.D. The Liberal Interpretation of Voting

Democrats of all persuasions would probably agree that participation built on the act of voting is the focus of democracy. But they certainly

interpret voting in different ways. What does it accomplish? What does it mean? The sharp dispute on these questions can be summarized in two views—one of which I call *liberal* or Madisonian, the other *populist* or Rousseauistic.[7]

In the liberal view, the function of voting is to control officials, *and no more*. Madison, who is the original American spokesman for liberal democracy (or republicanism, as he called it) defined a republic as "a government that derives all its powers directly or indirectly from the great body of the people, and is administered by people holding their offices during pleasure, for a limited period, as during good behavior."[8] The first requirement, popularness, he called essential (that is, *necessary*); the second, election and limited tenure, he called *sufficient*. Thus his definition is *logically* complete, and there is nothing to add. Madison said nothing about the quality of popular decision, whether good or bad.

Since all democrats would accept the necessary condition, it is the sufficient condition that is distinctive and hence deserving of detailed explication. Why is election and limited tenure sufficient? Popularness, the necessary condition, ensures participation and equality. The sufficient condition is intended to ensure liberty. In Madison's view, the danger for liberty lies in government officials who might deprive citizens of liberty or fail as agents of citizens' participation. In either case, the liberal remedy is the next election. That is all that is needed to protect liberty; so election and limited tenure are sufficient.

To consider first the protection of citizens' liberty: The replacement of officials is, in the liberal view, the only available instrument. The liberal fear is that the force of government can easily be deployed against citizens to make them support unpopular policies that officials believe necessary. The liberal hope is that officials will be restrained from such behavior out of fear of the next election. It is true that Madison and other framers of the Constitution provided the separation of powers as auxiliary protection, but Madison regarded that protection as distinctly secondary to "a dependence on the people." And the contemporary liberal agrees with Madison that the defense of liberty lies in the discipline of elections.

In the twentieth century it has sometimes (but not lately) been fashionable for populists to dismiss the liberal fear of oppression as an anachronism. Populists believe that, by reason of popular participation, democratic governments embody the will of the people and cannot therefore oppress. Only in the eighteenth century, they say, when executives were officers of the Crown was this danger real; now that elected executives supposedly embody the popular will, they cannot oppress. In *Roosevelt and Hopkins*, Robert Sherwood, for example, disputed Lord Acton's assertion that power corrupts with his (that is, Sherwood's) own belief

that power ennobled Franklin Roosevelt.[9] But it was Sherwood's other subject, Harry Hopkins, who presumably uttered that epitome of corruption: "We will tax and tax, spend and spend, elect and elect." Lately, of course, even populists have been shaken by the imperial presidency of Johnson and Nixon, who, however popularly elected, persisted in a hated and oppressive war. In both cases the threat of the next election proved decisive for liberty because it made one not try for reelection and the other (ultimately) end the war. Even more impressive, the possibility of impeachment, a kind of negative election, made Nixon resign. Moreover, it was elections themselves, not just the threat of them, that as recently as 1977 disposed of two putatively tyrannical rulers in India and Sri Lanka.

The other part of Madison's concern was a fear of tyranny by the majority. This is a fear that officials acting for a majority created in the last election will persecute the minority of that election. Madison hoped that such oppression would be minimized by the fact of shifting majorities, so that a future majority might throw out of office the officials who oppressed in the name of the former majority. This is the reason he stressed diversity in the electorate. The way, he said, "to guard one part of society against the injustice of the other part" is to comprehend "in the society so many separate descriptions of citizens as will render an unjust combination of a majority of the whole very improbable, if not impracticable."[10]

Viewed statically, this sounds like just another version of the separation of powers.[11] Viewed dynamically, however, this is simply the claim that an unjust majority cannot last through several elections. Looking at the oppression of blacks, the most persistent issue in American politics and the clearest case of tyranny by the majority, it appears that the Madisonian hope has been justified. As long as blacks were excluded from the political system (from the beginning to 1867 and from the end of Reconstruction in 1877 to the emergence of a substantial number of black voters in the 1930s), they were persecuted. But including them in the system, especially as they became a marginal bloc between the political parties, led to political reform and even to reorientation of the judiciary, so that national political leaders (followed by the courts) have mitigated and are gradually eliminating that tyranny by the majority.

To consider the other danger to liberty (that officials be inefficient agents): The only possible remedy—and one recommended by both populists and liberals—is to elect new officials. So again the next election promotes liberty. Notice, however, that in the liberal view it is not assumed that the electorate is right. This assumption characterizes populism, as I will show. The liberal assumes not popular competence, but merely that

the electorate can change officials if many people are dissatisfied or hope for better performance.

It may seem that in the liberal view officials, who are only negatively controlled by voting, cannot really act as agents of the electorate. By reason of regular elections, however, officials may be rejected. In their efforts to avoid rejection they usually act in some rough way as agents of the electorate, at least attempting to avoid giving offense to some future majority. Since this future majority cannot at any moment be clearly specified, officials seeking to placate it in advance must anticipate several kinds of potential majorities, the union of which is often most of the electorate. By reason of this anticipation of the next election, officials are, even in the liberal view, subject to electoral discipline as the agents of democratic self-control.

1.E. The Populist Interpretation of Voting

For the populist, liberty and hence self-control through participation are obtained by embodying the will of the people in the action of officials. The fundamental notion goes back at least to Rousseau. There is a social contract, which creates a "moral and collective body" that has "life" and "will," that is the famous "general will," the will of the incorporated people, the Sovereign. Individual liberty, for Rousseau and subsequent populists, is the participation of the citizen in this sovereignty. "Liberty," Rousseau says, "is obedience to a law we have prescribed for ourselves," understanding, of course, that the prescription is through the acts of the anthropomorphized Sovereign.[12] The way to discover the general will, which is the objectively correct common interest of the incorporated citizens, is to compute it by consulting the citizens. The computation will be accurate if each citizen, when giving an opinion or vote, considers and chooses only the common interest, not a personal or private interest. Thus, by summing the common interest regarding wills (votes) of real persons, one can arrive at the will of the great artificial person, the Sovereign.

In the Middle Ages it was sometimes (blasphemously) said that the voice of the people is the voice of God. Rosseau did not invest the people with quite such divine authority—indeed he believed they might be mistaken about the general will—but he did assert that the general will is always correct and embodies the objective good for society. Later populists have continued to attribute some special character to the voice of the

people: What the sovereign people, when speaking for the public interest, want is justified because the sovereign people want it and because it is their liberty.

To summarize: According to the populist interpretation of voting, participation in rule-making is necessary for liberty. The rules thus made must be respected as right and proper because they embody that liberty. Were they not so respected, liberty itself might vanish.

1.F. Differences Between Liberal and Populist Interpretations of Voting

In the liberal view, since voting generates liberty simply by restraining officials (by popular election and limited tenure), there is no need to treat the output of government as the precious embodiment of liberty itself. Indeed, for the liberal, law is no more than the decree of legislators or judges, accepted and occasionally ratified by the citizens. But in the populist view, since voting generates liberty by participation, the output of government must be precious, for that very output *is* liberty.

We can understand the difference between the two views somewhat better, I believe, by recounting a controversy over Isaiah Berlin's distinction between positive and negative liberty.[13] Berlin defined *negative liberty* as the absence of interference by others (especially government) in one's activity. *Positive liberty,* on the other hand, is being one's own mentor. The burden of Berlin's argument is that these two apparently similar notions are at loggerheads. Berlin's explanation of this paradox is that ideas about self-mastery are turned into particular goals for society that people are then coerced into following: They are forced, that is, to be "free." His main example is the transformation of Kant's notions of individual ethical responsibility by, successively, Hegel and Marx into the justification for a monstrous dictatorship. Berlin's point is that positive liberty, which appears initially innocuous, is the root of tyranny.

C. B. MacPherson, in a clever effort to rescue positive liberty from Berlin's obloquy, redefined three kinds of positive liberty:

1. *Liberty as "self-direction" or "self-mastery":* "to live in accordance with one's own conscious purposes."
2. *Liberty as obedience to law:* "coercion, by the [supposedly] fully rational or those who have [supposedly] attained self-mastery [e.g., Lenin or Mao], of all the rest."

3. *Liberty as participation:* "the democratic concept of liberty as a share in the controlling authority." (Notice that this is the populist, not the liberal view of voting.)[14]

MacPherson's argument is that Berlin, lumping the three kinds of positive liberty together, used the obvious and admitted difference between negative liberty and populist voting to generate a contrast and (through liberty as obedience and coercion) an inconsistency between negative liberty and all kinds of positive liberty. MacPherson, as a populist and socialist, wants to save self-mastery as complementary to and not inconsistent with negative liberty, while banishing coercion, that Stalinist embarrassment to "democratic" socialists. MacPherson accomplishes his purpose by giving self-mastery a new name, "developmental liberty," so that it appears to be clearly separate from liberty as obedience and coercion.

The problem, however, is that populist voting is ineradicably different from negative liberty and yet is logically correlated with coercion. This association is precisely explained by Willmoore Kendall in his explication of Locke's version of majority rule.[15] Kendall sought to reconcile Locke's presumed belief in objective moral law with his conclusion that right is what the majority wills. The explanation Kendall proposes of this apparent paradox is that Locke assumed that most people are just and rational; consequently, "The individual can . . . covenant to obey the majority without subjecting himself to . . . arbitrary authority . . . since the judgments of the majority are those of reason and justice."[16]

Thus it is that MacPherson's populist voting unites self-mastery and coercion. All one has to do is to find that a majority (perhaps a putative or even a wholly imaginary and nonexistent majority like the "proletariat" conjured up by Marx) has willed some version of self-mastery. It then becomes both *reasonable* and *necessary* to impose that version of liberty by coercion. It is *reasonable* because the majority that produced the particular version of self-mastery is, in Kendall's words, "rational and just." And it is *necessary* because the particular self-mastery is the embodiment of that majority's liberty, and its liberty would vanish were it not translated into a coerced version.

If, however, one had not the populist view of voting, but merely the liberal view, then this totalitarian sleight-of-hand would not be possible. Indeed, if there were only the liberal view and if one banished the populist "share in the governing authority," then it would be easy for MacPherson to demonstrate the complementarity between negative liberty and self-mastery. But, as a socialist, MacPherson cannot give up populism, and so

he is necessarily stuck with coercion. Clearly, Berlin's villain all along is really populism, not just self-mastery. He should have contrasted not negative and positive liberty, but rather negative liberty and the populist view of voting that is used to justify coercion in the name of temporary or spurious majorities.

I have emphasized the moral certainty implicit in the populist view of voting in order to correct the common misconception that populist democracy is simply majority rule. The customary distinction—often expressed in the cliché "majority rule versus minority rights"—is between (1) popular sovereignty or lodging power and decision in the hands of the majority and (2) limited government or reserving some rights for minorities (within an otherwise majoritarian framework). Dahl, for example, makes this distinction, in effect terming (1) Madisonian and (2) populist. This distinction cannot be maintained, however. All democracies involve popular government, equality, and the rule of decision according to the greater number—precisely the features Dahl attributes to populism. Conversely, all democracies (populist as well as liberal) actually limit government by the technique of shifting majorities. So the customary distinction is without a difference.

What is different between the liberal and the populist views is that, in the populist interpretation of voting, the opinions of the majority *must* be right and *must* be respected because the will of the people is the liberty of the people. In the liberal interpretation, there is no such magical identification. The outcome of voting is just a decision and has no special moral character.[17]

1.G. The Vitality of the Liberal and Populist Interpretations of Voting

To show that two interpretations of voting have existed does not prove that they continue today to influence thought. They may have been amalgamated, and if so there is not much point to discussing them, except as historical phenomena. Two writers I have quoted—Berlin (fatalistically) and MacPherson (hopefully)—have each tried to show that populism is absorbing liberalism. I believe, however, that the two traditions remain separate. My evidence is two recent books whose authors are deeply concerned in a practical way with changing American politics, who do not bother with abstract political philosophy, but who nevertheless reflect in more or less pure form the two interpretations I have described.

The books are William A. Rusher's *The Making of the New Majority Party,* which is the liberal offering, and Marcus Raskin's *Notes on the Old System,* which is the populist offering.[18]

Both writers were impelled to write by the Watergate crisis. Rusher observed the scandal weakening the Republican party at the very moment that a majority of citizens seemed to him to be abandoning the then-dominant statism for so-called conservatism (actually Madisonian liberalism). These citizens lacked, he feared, an appropriate vehicle through which to express their new values, not only because the Republican party was discredited, but also because it was infected with the statism of, for example, Nelson Rockefeller. Consequently, Rusher wrote to propose a new national party that would organize the now-leaderless putative majority.

Raskin, on the other hand, regarded Watergate as another compelling instance of the way that what he called "the System" (in which he included the imperial presidency, the CIA, rich people, the Joint Chiefs, the Democratic party, and capitalism) frustrated the supposed impulses of most people to take the positions he (Raskin) believed correct on public policy. Seizing the occasion, therefore, he wrote both to induce popular disgust and to propose a reform—namely, a nationwide system of grand juries to instruct members of Congress. This change, he believed, would amplify voices now supposedly muffled.

Both authors describe themselves as democrats, accurately, I believe, although Raskin displays tendencies toward MacPherson's preference for coercive liberty. Both Rusher and Raskin also denounce the huge bureaucratic apparatus of the contemporary state and seek to hook this Leviathan. But there the similarity ends, and each proceeds according to the tradition he represents.

Rusher thinks the Leviathan exists in large part because voters are often wrong, misled by demagogues who promise "benefits" that cannot be paid for and will, by inducing inflation, harm the ostensible beneficiaries. Clearly, he has no populist illusions that the people do what is right. On the other hand, he wants to make democracy work and rejects the possibility of limiting suffrage. Instead he proposes a new party to instruct and lead the people. "How," Rusher asks, "does an honest politician . . . run against some spellbinder who has invented a new 'benefit'?" The answer: "By telling the truth, of course, about the real cost and impact of the proposed 'benefit'" (p. 200). This answer is introduced by quoting Madison, who wrote that "knowledge will forever govern ignorance" and popular governments "must arm themselves with the power knowledge gives" (p. 198). Thus, Rusher's prescriptions are exactly liberal: Without

supposing the people are especially wise, one should nevertheless try, at the polls at the next election, to hook the Leviathan, the king of the children of pride.

Raskin thinks that the Leviathan exists because an elite (politicians, bureaucrats, soldiers, corporate executives) uses the System to suppress the voice of the people and do all sorts of wrong things like building MIRVs and collecting withholding taxes. He is confident that the people, if they could speak, would do right. Like Rousseau, he wants to erase the special interests that stand between the people and the general will. His method is a huge number of grand juries to conduct inquiries and to instruct members of Congress. A Congress thus revitalized would, he believes, embody the true will of the people in law.

Raskin's scheme contains both features of the populist view. First, it stresses participation in local assemblies that will be "instruments of the people as people and as citizens" (p. 157). Second, what the people do will be good, for they will "express a quality of empathy, fairness, and inquiry" (p. 152). Raskin quotes approvingly these sentences from Martin Buber:

Though something of righteousness may become evident in the life of the individual, righteousness itself can only become wholly visible in the structure of the life of a people. . . . Only life can demonstrate an absolute, and it must be the life of the people as a whole.[19]

As applied by Raskin, this is the most extreme claim of populism I have ever seen. Apparently, Raskin believes that the works of the people embody not only their liberty and true justice, but also (and incredibly) *absolute* righteousness.

1.H. The Paradox of Voting

As a beginning of the discussion of the attainability of democratic justice, I have so far shown that the ideal as well as the method of democracy focuses on voting and that voting has been interpreted in two quite different ways. Now I want to show that the theory of social choice raises other disturbing questions about voting, questions that are as controversial as the issue between liberalism and populism and that may affect or even resolve that issue.

Historically, the theory of social choice arose out of the paradox of

voting. Without here going into many subsequently developed niceties, one can explain the paradox with these primitive notions of preference and choice.

1. *Preference.* Assuming there are *alternatives,* $x, y, \ldots$, which may be objects, values, motions, candidates, and so on, a person, i, may prefer one alternative to another. This state of mind is represented as the relation of *preference,* P, between some pair of alternatives, x and y. Conventionally, one writes $x\ P_i\ y$, to mean "i prefers x to y." The relation, P, is *transitive,* which means that the following sentence is true:

   If $x\ P_i\ y$ and $y\ P_i\ z$, then $x\ P_i\ z$.

   Quantitative relations like equality ($=$) or greater than ($>$) are transitive. For example, if a equals b and b equals c, then a equals c. Other relations, such as parenthood, are not transitive. Clearly, if a is the father of b and b is the father of c, then it does *not* follow that a is the father of c. Preference is said to be transitive mainly because intransitive preferences usually seem bizarre. If a man says he likes Republicans better than Democrats and Democrats better than Communists, then we think he is indeed strange if he also says he likes Communists better than Republicans.

2. *Rules of Choice.* Given a society of n persons, where i is one individual, and given a set of alternatives, $X = (x, y, \ldots)$, a rule of choice is a rule by which a choice, C, is made for all of the n persons (e.g., the selection of a winning alternative by voting or the selection of the alternative left after discussion has eliminated all expressions of dissent). Conventionally one writes $C(X) = y$ to mean "the social choice from X is y."

   There are many rules of choice. A typical example is *simple majority voting between two alternatives.* By this rule, if more people prefer x to y than prefer y to x, then x wins. Conversely, if more people prefer y to x than x to y, then y wins. And if the same number prefer x to y as prefer y to x, then x and y tie.

With these primitive notions of preference and choice, the paradox of voting can now be stated: Suppose three people, 1, 2, 3, choose among three alternatives, x, y, z, by the method of simple majority rule applied successively to pairs. Suppose also that each person has the following transitive ordering of preference on x, y, and z:

Person 1: $x P_1 y$, $y P_1 z$, and $x P_1 z$; or $x\, y\, z$

Person 2: $y P_2 z$, $z P_2 x$, and $y P_2 x$; or $y\, z\, x$

Person 3: $z P_3 x$, $x P_3 y$, and $z P_3 y$; or $z\, x\, y$

Then the social choices are:

$C(x, y) = x$ because $x P_1 y$ and $x P_3 y$, while $y P_2 x$

$C(y, z) = y$ because $y P_1 z$ and $y P_2 z$, while $z P_3 y$

$C(x, z) = z$ because $z P_2 x$ and $z P_3 x$, while $x P_1 z$

Thus, although each individual in the society has a transitive ordering of preference, the outcome of voting is not transitive because x beats y, y beats z, and z beats x. If one tried to arrange the outcome of voting in a sequence of "social preference," one would not be able to do so because one could not say whether x or y or z stood first. Any one of these arrangements would be possible: $x\, y\, z\, x$, $y\, z\, x\, y$, $z\, x\, y\, z$.

If, on the other hand, one imposed transitivity by starting with $C(x, y) = x$ *and* $C(y, z) = y$ *and* concluding, by reason of transitivity, that $C(x, z) = x$, then person 1 would be a dictator, because only person 1 prefers x to z. Apparently, one is forced either to accept intransitivity for society or to achieve transitivity at the cost of creating a kind of dictator.

Many people are shocked by this result. One standard of consistency in sentences and coherence in thought is transitivity. We would consider a person claiming to like five dollars more than three dollars, three dollars more than one dollar, and one dollar more than five dollars to be quite confused. So we say that *preference* is a transitive relation. We can go further and say that social choice should also be transitive. If so, then, in the case of the voting paradox, we must affirm, paraphrasing Reinhold Niebuhr, "coherent man and incoherent society." Although individuals can arrive at a unique choice, in this case society cannot even choose. What makes all this so democratically unpalatable is that, apparently, the only way to make "society" choose coherently is to impose a dictator.

The possibility that social choice by voting produces inconsistent results raises deep questions about democracy. Can the democratic ideal be attained if the method used to attain it produces confusion? Given the possibility of inconsistency, does one interpretation of voting make more sense than another, or are both interpretations hopelessly flawed?

To raise the issues in the bluntest possible ways, I ask:

1. Can voting restrain officials if the outcome of voting is inconsistent? How can restraint occur if it is not clear what restraint is imposed?

2. Is someone, supposedly restrained at the polls, merely kept in or out of office accidentally? If the outcome of voting might be $x\,y\,z\,x$ or $y\,z\,x\,y$ or $z\,x\,y\,z$, does not an accident of institutions, rather than popular taste, select the winner? And if the antecedent constitution, rather than the people, chooses, how can any kind of democracy be said to operate?

3. If liberty is embodied in an inconsistent law, is not liberty itself defective? If alternative laws are in a cycle, $x\,y\,z\,x$, then which alternative ought to be regarded as the will of the people and their liberty?

4. When an absolute good produced by voting is inconsistent, can that absolute have any moral significance? To say that x is morally right because x beat y seems difficult to defend if z also beat x.

These are the kinds of questions raised when we allow the theory of social choice to confront the theory of democracy. In this book I will elaborate the theory of social choice in order ultimately to explore these questions.

3

Simple Majority Decision

The examples of Chapter 2 clearly indicate that different methods of summarizing the same underlying structure of preference produce different social choices. Still, this fact need not cause problems for democratic theory if one method of aggregation is clearly superior to all others. And, as it happens, there is one that is quite attractive and perhaps appears better than others—namely, simple majority decision when there are exactly two alternatives.

The procedure does have highly desirable properties, and one reason that most people are indifferent to the defects in voting may be that they believe that simple majority rule over two alternatives is in use most of the time. If this belief were correct, popular indifference would probably be justified. Unfortunately, simple majority rule displays its most desirable properties only when there are exactly two alternatives, and nature hardly ever offers us binary choices. We have, of course, many institutions, like primary elections, to reduce alternatives to exactly two. But simply because we force ourselves into a binary choice should not obscure the fact that we really start out with many alternatives and that we can never be certain that our institutions have narrowed the choice down to the right pair for us to choose between.

Rare as its natural occurrence is, however, simple majority decision is still very attractive, which is why some people want to realize it in institutions. In this chapter, therefore, I will explain in detail why it is attractive. Then, in conclusion, I will describe the tradition of democratic thought devoted to institutionalizing it. But I will also show that simple majority decision cannot be institutionalized without violating fundamental notions of fairness.

In this chapter and the next two, it will be necessary to define a number of technical terms in order to reveal some of the deep meaning of

the issues raised. Definitions will always be stated in the text. Formal statements—if useful—will be placed in the Notes. Since this is an introductory discussion, no proofs of formal theorems will be given.

3.A. Introductory Definitions

In section 1.H several primitive notions were introduced. I will elaborate them here and introduce a few more.[1]

1. A set, $X = (x, y, \ldots)$, of alternatives, which can be motions, candidates, platforms, bundles of goods, and so on. Since in this chapter the concern is with simple majority choice between pairs, the set X will here be limited to exactly two members, $X = (x, y)$; later it may be indefinitely or even infinitely large. Here, x and y may be thought of as two candidates (x is, say, candidate A, and y is then candidate B) or as yea or nay votes on a motion (x is for yea, and y is for nay).

2. A set, $N = (1, 2, \ldots, n)$, of people, which is a society of n members or eligible participants, where i is any one of the n members, that is, $i = 1$ or 2 or $\ldots$ or n.

3. For each i, a relation, P_i, of preference over X. Each P_i is asymmetric: If a judgment is made between x and y, then either $x\,P_i\,y$ or $y\,P_i\,x$ but not both. Notice that P_i need not be complete so that, for some x and y in X, neither $x\,P_i\,y$ nor $y\,P_i\,x$.

4. For each i, a relation, I_i, of indifference, which, given a judgment between x and y, is defined as: not $(x\,P_i\,y)$ and not $(y\,P_i\,x)$. Hence I is symmetric: if $x\,I_i\,y$, then $y\,I_i\,x$.

5. For each i, a relation, R_i, of P_i and I_i combined, which is defined as: $x\,R_i\,y$ is equivalent to not $(y\,P_i\,x)$.[2] Notice that R_i is complete so that either $x\,R_i\,y$ or $y\,R_i\,x$.

6. The concept of a social choice, $C(X)$, where $C(X) = z$ means "When the society follows a given rule for choosing, the choice from X is z." If the context does not make clear the rule being followed, say, simple majority (SM) or plurality (P), then the rule in use will be indicated as a subscript to C, thus: $C_{SM}(X)$ or $C_P(X)$.

It is a delicate matter to determine just what P_i, I_i, and R_i involve. One possible interpretation is that they refer to private individual judgments. We learn of such judgments, however, only through their public

expression, as when a person says, "I like y better than x," or when a person votes for x. Unfortunately, public expressions may or may not truly reflect private judgments. A person may dissemble opinions, for strategic reasons, and perhaps vote contrary to true tastes (either because of being manipulated or because of trying to manipulate). It is not quite clear, therefore, just what the statement $x\ P_i\ y$ means. For convenience I will adopt this convention: Except when I expressly note that private judgment and public action may differ, I shall equate the two. Hence $x\ P_i\ y$ will mean both private preference for x over y and the public (though possibly secret) act of voting for x, and $x\ I_i\ y$ will mean both private indifference and abstention from voting.

For each i in N, there is a structure, D_i, of individual judgment over the members of X. In the two-alternative case, D_i is just one of $x\ P_i\ y$, $x\ I_i\ y$, or $y\ P_i\ x$. These forms can be represented conveniently by the integers $(1, 0, -1)$:

$$\begin{aligned} D_i = 1 \quad &\text{means} \quad x\ P_i\ y \\ D_i = 0 \quad &\text{means} \quad x\ I_i\ y \\ D_i = -1 \quad &\text{means} \quad y\ P_i\ x \end{aligned} \tag{3-1}$$

For the whole set N, there is a vector, D, of all the individual judgments, where $D = (D_1, D_2, \ldots, D_n)$. Using (3-1), some profile, D, of a society may be thought of as a string of integers chosen from $(1, 0, -1)$, where each integer stands for an individual judgment. For example, on a five-person committee, where the first, fourth, and fifth members favor and vote for a motion, the second is undecided and abstains, and the third opposes and votes against, $D = (1, 0, -1, 1, 1)$. Finally, the vector D is a member of a set $\boldsymbol{D}$, which is the set of all possible social profiles. For example, if N consists of two people (that is, $n = 2$), then $\boldsymbol{D}$ is as set forth in Display 3-1.

Thus, D_i is some particular person's preference or values, D is the profile on a particular occasion of the preference or value structure of all members of the society, N, and $\boldsymbol{D}$ is the set of all possible social profiles that N might display.

Inasmuch as our concern is with some kind of *social* choice, based ultimately on individual judgment, there must be a rule by which the D_i in each D are amalgamated. This rule is a function, F, which is stated in complete form so that it can operate on any D in $\boldsymbol{D}$. The social choice from X, given D and F, is referred to as $F(X, D)$. I will abbreviate this to $F(D)$. Note that $F(D)$ is thus a special case of $C(X)$, special in the sense that a profile, D, and a rule, F, are stipulated. For the case where X has

Display 3-1

Members of *D* for Two Persons

$$\boldsymbol{D} = \begin{Bmatrix} (1, 1), (0, 1), (-1, 1) \\ (1, 0), (0, 0), (-1, 0) \\ (1, -1), (0, -1), (-1, -1) \end{Bmatrix}$$

Each pair of D_i in parentheses indicates a possible choice by the two voters. Thus (0, 1) means that $D_1 = 0$ (for $x\ I_1\ y$) and $D_2 = 1$ (for $x\ P_2\ y$). The set of nine possibilities in large brackets is all possibilities of $\boldsymbol{D}$.

just two members, I define $F(D)$ parallel to the interpretation of D_i:

$F(D) = 1$ means x is the social choice from X by F, given D.

$F(D) = 0$ means x and y tie and are together the social choice by F from X, given D. (Many rules of amalgamation do not exclude ties. Practically, however, there are usually ad hoc rules to break ties—tossing coins, extra votes for presiding officers, etc.—which have nothing to do with the amalgamation itself.)

$F(D) = -1$ means y is the social choice by F from X, given D. (3-2)

Obviously, there are a large number of possible rules or functions. To convey a sense of the wide variety, I will list a few to be discussed:

1. *Constant functions*
   a. *Indecisive.* x and y always tie, regardless of the individuals' preferences in D; that is, $F(D) = 0$, for all D in $\boldsymbol{D}$.
   b. *Imposed.* x (or y) always wins, regardless of the individuals' preferences in D; that is, $F(D) = 1$ (or -1), for all D in $\boldsymbol{D}$.
2. *Simple majority functions.* That alternative wins which has more votes than the other (or, if the number of voters is even, alternatives may tie), where members of N may vote or abstain. Many variations can be obtained by weighting votes and declaring the winner to be that alternative with the greater sum of weights.

3. *Absolute majority functions.* That alternative wins which has more votes (or weights) than half of the total votes or weights in N, with abstention permitted or not permitted as the case may be.

4. *Special majority functions.* That alternative wins which has more than some specified proportion of the votes or weights (e.g., ratios like 2/3 or 3/4), when the proportion is calculated from those voting or from all members of N.

3.B. Properties of Simple Majority Decision: Monotonicity

The method of simple majority decision, in which voters are weighted equally and in which they can either vote or abstain, is defined thus: If more people vote for x than for y, x wins; if the same number vote for x and y, they tie; and if more vote for y than for x, y wins. Using the terminology of D_i, if the sum of the D_i in D is greater than zero, x wins; if equal to zero, x and y tie; and if less than zero, y wins.[3] The method has three independent properties—monotonicity, undifferentiatedness, and neutrality. An examination of each of them will permit a full appreciation of this rule.[4]

By *monotonicity* is meant that an increase in the value of some D_i implies an increase, or at least not a decrease, in the value of $F(D)$. That is, if one or more voters change preference in a direction favorable to x (i.e., change from favoring y either to being indifferent between x and y or to favoring x, or change from being indifferent to favoring x), then the resulting change, if any, in the fate of x should be an improvement for x.

Monotonicity is an especially important feature of any decision rule that amalgamates individual tastes into a social outcome. It would be perverse in the extreme if increased votes for an alternative contributed to its defeat. Consequently, it seems an elementary requirement of sensible and fair choice that the decision rule respond positively, or at least non-negatively, to increases in individual valuation of an alternative. This is precisely what monotonicity (or, as it is sometimes called, *responsiveness* or *nonperversity*) provides for. Desirable as monotonicity may seem, however, there are, quite surprisingly, a number of widely used social choice functions that fail to satisfy it.

To define monotonicity, consider some D and D' both in $\boldsymbol{D}$. They may be identical; but for every i in N, $D_i \geq D'_i$. Specifically, any change from D' to D involves a change, for one or more i, in the direction of fa-

voring x over y. Thus, for any person, i, whose values change, if $D_i' = -1$ (i.e., $y P_i x$), then $D_i = 0$ or 1; or if $D_i' = 0$ (i.e., $x I_i y$), then $D_i = 1$. This means that, for the whole society, the sum of D_i is equal to or greater than the sum of D_i', because, if any change occurs from D_i' to D_i, it must increase the sum of D_i relative to D_i'. Hence monotonicity for a rule, F, means that if some D_i increases over D_i', then $F(D)$ is not less than $F(D')$.[5]

To state this definition in another way, suppose an alternative is rendered more preferred in private judgment and individual voting. Then, if any change in outcome results, that change *must* not hurt the alternative that has come to be more attractive. But, of course, no change in outcome need result. Indeed, this condition on F does allow for a wide range of ties and hence for a wide range of situations in which, despite shifts in preference or valuation, no change in outcome results. Specifically it might happen, for an F satisfying equation (N3.2) (see note 5),* that some person changes $D_i' = 0$ to $D_i = 1$, yet nevertheless $F(D') = F(D) = 0$. This means that an individual valuation of x rises, but the tie between x and y is not broken.

A commonly used method of social choice that displays such a wide range of ties is the rule used on juries. For a jury, where x means conviction and y means acquittal, the jury function is the rule that x or y wins only if the jury is unanimous and that otherwise x and y tie in a hung jury.[6] For this F in situation D' the jury might be split nine to three for conviction (i.e., $\Sigma_{i=1}^{n} D_i' = 6$). Then to create situation D some juror, i, might switch from acquittal to conviction so that $D_i' = -1$ and $D_i = 1$. (Consequently, $\Sigma_{i=1}^{n} D_i = 8$.) But the jury would still be hung, so $F(D') = F(D) = 0$.

In simple majority decision, ties are not so persistent, which is one of the merits, I believe, of this rule for large electorates. To describe simple majority decision, therefore, one needs the notion of *strong monotonicity:* If a tie exists and just one voter shifts his or her position, the tie is broken.[7] Thus, if just one person shifts from abstaining ($D_i' = 0$) to voting ($D_i = 1$ or -1), strong monotonicity requires that the tie be broken, provided no one else changes a vote. On, for example, a seven-member committee with six members present and split three to three on election of a chairman, the absent member will, when brought in to vote, break the tie.

As this example indicates, simple majority voting is strongly monotonic because a single person can always break a tie if no one else changes. The jury rule, while monotonic, is not strongly monotonic, because changes by several persons may be required to break a tie.

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*The prefix "N" indicates that the equation appears in the Notes at the back of the book.

Pages 47-50 are omitted.

3.D. Properties of Simple Majority Decision: Undifferentiatedness (Anonymity)

As we turn now to other properties of simple majority decision, moral rather than technical considerations dominate. The property of undifferentiatedness (or anonymity, as it is usually called) is imposed primarily because of preferences about political values, although the condition itself is technical.

Undifferentiatedness is often said to be the same as equality and to embody the principle of "one man, one vote." Actually, however, it is the technical condition underlying equality and is quite distinct from equality. The best name for the condition is *undifferentiatedness* because this is primarily what it provides—namely, that one vote cannot be distinguished from another. This feature allows for anonymity, and anonymity in turn allows for equality. In the next few paragraphs, I will explain the sequence from technical antecedent to moral consequent.

Sometimes voters are clearly distinguished by the differentiated votes they cast. They may, for example, be assigned unequal numbers of votes. These may be thought of as weights, w_i, where $i = 1, 2, \ldots, n$. On

the New York City Board of Estimate, the mayor, comptroller, and president of the City Council each have four votes, while the five borough presidents have two votes each. In effect, each official's judgment on a motion is multiplied by the number of assigned votes. This fact can be described by writing $D = (w_1D_1, w_2D_2, \ldots, w_nD_n)$, where $w_i \geq 0$. Calculating the sum of the weights, W, the rule for weighted voting is: That alternative wins which receives more than half of the sum of the weighted votes; and, if both alternatives get the same weighted votes, they tie.[13]

Naturally, votes must be differentiated from each other so that vote-counting will be accurate. One possible formulation of the condition of undifferentiatedness is, therefore, that weights be equal: $(w_1, w_2, \ldots, w_n) = (1, 1, \ldots, 1)$. This formulation is inadequate, however, because votes are also differentiated by the roles the voters play in the system. For example, in the United Nations Security Council, for a (substantive) motion to pass, all five permanent members (US, USSR, China, Britain, France) must vote yea and so must four of the ten temporary members. The permanent members' rights of veto clearly require that their votes be differentiated. Several scholars have translated rules for this sort of differentiation into weights in a weighted voting system.[14] Unfortunately, the methods of translation vary in assumptions and therefore disagree in results, sometimes wildly.[15] It seems to me wise, therefore, not to try to reduce all differentation to a matter of unequal weights. Since roles as well as weights require that votes be differentiated, it is essential that the definition of undifferentiatedness capture the fact that neither makes a difference.

To do so, I introduce the notion of a *permutation.* Let $(1, 2, \ldots, n)$ be a sequential arrangement of n objects. One can rearrange them by replacing the first object in the sequence with another object (including possibly itself), the second object with another, and so forth through the n^{th} object. If we signify the replacement itself with $\sigma_{(i)}$ where σ indicates the replacement and the subscript indicates the object replaced, a permutation of $(1, 2, \ldots, n)$ is then a new arrangement: $(\sigma_{(1)}, \sigma_{(2)}, \ldots, \sigma_{(n)})$. For example, if the initial arrangement is $(1, 2, 3)$ and $\sigma_{(1)} = 2$, $\sigma_{(2)} = 1$, and $\sigma_{(3)} = 3$, then the permutation is $(2, 1, 3)$.

Undifferentiatedness is the condition that any permutation of a set of individual judgments leads to the same social choice.[16] This means that the votes cannot be differentiated either in weight or in the roles played by the voters because if judgments are rearranged among voters *in any way* the same outcome is produced. Thus, for example, on a five-member committee, undifferentiatedness requires $F(1, 0, -1, 1, 1) = F(-1, 1, 1, 0, 1)$. If we use simple majority rule, $F(D) = 1$ for both these sequences.

Undifferentiatedness provides the technical base for anonymity but is not the same thing. Votes are undifferentiated, while voters are anonymous. Suppose in some society undifferentiated votes are bought and sold. Buyers do not care which votes they buy because votes are undifferentiated. Hence the purchase of k individual judgments in one permutation produces the same outcome as the purchase of k votes in another permutation. Thus the definition of undifferentiatedness (which is also the usual definition of anonymity) is satisfied. Nevertheless, in that society voters cannot be anonymous because their names are necessary for enforcement of contracts.

Once votes are undifferentiated, however, it is possible to detach names, and historically this is precisely what happened. In the Anglo-American tradition, the content of legally undifferentiated votes was, in the eighteenth and early nineteenth centuries, recorded by the name of the voters in poll books. Since this practice was believed to admit both corruption and coercion of voters, it gradually gave way to printed ballots. Since the ballots were produced and distributed by party workers, they knew how voters voted, and corruption and coercion were still possible—and indeed widespread. Finally, the secret ballot, a late-nineteenth-century invention, effectively provided anonymity. Notice, however, that it was historically a necessary condition of anonymity that votes be undifferentiated.

The same development from undifferentiatedness to anonymity occurred in a curious way in ancient Athens. As the selection of officials by heredity gave way to election by legally undifferentiated voters, political leaders developed groups of clients whose votes they coerced. Two devices were then developed to provide anonymity: Most officials were selected by a lottery, and the size of juries, which decided many political questions, was increased in some cases to sixteen hundred jurors whose votes were, presumably, hard to supervise.

The motive for providing anonymity for persons who cast otherwise undifferentiated votes is a belief in the ethical principle of equality, which is especially attractive to leaders who have relatively few clients. Those who cast undifferentiated votes may not be equal, for the operative principle can be "one important man, many (clients') votes; one client, no personally decided vote." Anonymity permits this rule to change to "one man, one vote," because all voters can then have an equal personal impact on the outcome (assuming monotonicity). Hence it is the ideal of equality that (morally) justifies anonymity, and it is the fact of anonymity that admits the practice of equality.

Whether it is desirable to impose undifferentiatedness and, further, anonymity on a method of social choice depends on whether one wants

to achieve equality of influence on outcomes. In business corporations, where, presumably, influence on outcomes is roughly proportional to ownership, neither anonymity nor differentiation is desired. In representative bodies, legislators are expected to be legally undifferentiated but not anonymous. Citizens need to know how their representatives vote. Furthermore, in the operation of party government, legal undifferentiation is replaced by de facto extra weighting of party leaders by giving them the weights of disciplined backbenchers. Thus, party government requires that nonanonymous legislators have identifiable (nonanonymous) votes; but in order to guarantee equality of representation, these identifiable votes must be undifferentiated. On the other hand, for popular elections of government officials it is usually desired that voters be treated equally, and for this, undifferentiated votes must also be cast by anonymous persons.

To illustrate the significance of undifferentiatedness for equality, I will conclude with a comparison and contrast of a system intended to promote inequality—the so-called demand-revealing process that Tideman and Tullock have recently proposed to extend to ordinary elections.[17] This process was invented as a device to motivate truthful statements of the demand for public goods such as schools and armies. It operates like this when applied for more than two voters on a binary choice: Each voter, i, offers to pay money, m_i, to obtain a preferred alternative, x or y. The amounts for x are summed over all who offer to pay for x into an S_x, and similarly for y into an S_y. The alternative with the largest sum wins. To motivate truthfulness, each person, i, on the winning side, say x, who offers to pay more than the margin of winning—that is, when $m_i > (S_x - S_y)$—must pay a tax of the amount of his or her contribution to the victory of x, which is $[S_y - (S_x - m_i)]$, when that number is positive. This tax is then destroyed. For example, suppose there are voters who offer the amounts shown in Display 3-4.

Since $S_x > S_y$, x wins. Person 2 must pay $[S_y - (S_x - m_2)] = 10 - (15 - 12) = \$7$, which is destroyed. (Were the \$7 given to person 1 and person 4 or used to run government, the losers would have a motive to bid up to \$14.99, which would increase taxes on persons 2 and 3 to \$14.98.) On the other hand, since $[S_y - (S_x - m_3)]$ is negative (i.e., $10 - (15 - 3) = -2$), person 3 pays no tax since x would win without his or her vote.

A voter in this process who offers more than his or her true valuation on an alternative may have to pay a tax higher than his or her true valuation. For example, if persons 1 and 4 each offered \$8 to make y win, they would each have to pay a \$7 tax, which is more than y is really worth to them. A voter who offers less than his or her true valuation risks losing.

Display 3-4

The Demand-Revealing Process for Two Alternatives

Money Offers, m_i, for Alternatives x and y

	x	y
Person 1	—	\$ 5
Person 2	\$12	—
Person 3	\$ 3	—
Person 4	—	\$ 5
Total	S_x = \$15	S_y = \$10

In either case, he or she is somewhat motivated to tell the exact truth, if it is known.[18]

As Tideman and Tullock point out, this process is like simple majority rule with secret ballots in that an abstainer gets what the higher bidders want, except when the abstainer decides to vote and can change the outcome and thus get what he or she wants. There the similarity of procedures ends, although they share the goal of truthful revelation. By treating voters anonymously, simple majority decision by secret ballot is intended to minimize coercion and corruption and thus allow voters to express their true preferences, although they may misrepresent their preferences if they find doing so to their advantage. Demand-revelation is also aimed at revealing true preferences, but it operates by treating voters unequally and requiring public revelation. The two systems are thus at loggerheads, and the difference between them is the moral value placed on equality.

It seems to me that the demand-revealing process promotes inequality in two ways:

1. Since utilities for money differ and since therefore more prosperous people (other than misers) are more likely to offer large amounts than are less prosperous people, demand-revelation clearly gives an advantage to the better-off. Indeed it might be described as a method to make wealth count for more.

2. Demand-revelation admits and encourages coercion because the winners' tax depends on the size of the losers' bids, which are necessarily revealed. (As pointed out in note 18, the tax is zero when $S_x > S_y$ and, for all i who have $x P_i y$, $m_i > S_y$. Surely this is an invitation to intimidation and coercion of exactly the sort that the secret ballot was intended to minimize.)

The strength of these forces toward inequality emphasizes by contrast just how deeply undifferentiated and anonymous voting provides for and is justified in terms of the notion of equality. Observe that item 1 violates the criterion of undifferentiated votes and that item 2 violates the criterion of anonymous voters. This joint violation renders equality impossible and thus reveals how equality is built on the sequential layers of undifferentiatedness and anonymity.

3.E. Properties of Simple Majority Decision: Neutrality

The third condition of simple majority decision is *neutrality* (or, as it is also known, *duality*), which means that the method does not favor either alternative. Many decision rules give an advantage to the status quo or to expediting motions in a committee (and there are often good reasons to do so). By contrast, the condition of neutrality, which seems especially appropriate for contests between candidates, provides that neither has an advantage.

One way of characterizing this idea is to note that, if neither alternative has an advantage, reversed preferences will lead to a reversed result. This statement is not generally true of rules that favor one alternative. For example, under the two-thirds rule, where x stands for the yea side, y is advantaged because it can win with no more than one-third plus 1 of the vote. Suppose y does win minimally and then the individual judgments are reversed so that x has one-third plus 1 of the vote. Still y wins. Under simple majority rule, however, if x wins minimally with just over half of the vote and preferences are reversed, y will have just over half and win.[19]

One important feature of neutrality is that decision rules embodying this condition typically allow for ties. Consequently this condition is inappropriate when decisions must be made. *Decisive* rules, which do not admit ties and are not neutral, have therefore a role in social choice. Here are some examples of decisive rules:

1. *Simple majority decision with a rule for breaking ties.* Ordinarily, under simple majority rule, neither side wins if the sum of D_i is zero. But if there is a rule to pick a winner, then that alternative has an advantage and the procedure is not neutral. Here are two examples of commonly used winner-picking rules (with their rationales, some of which appear dubious):
   a. In a legislature, when yeas and nays tie, nays win. (Rationale: In the absence of a clear majority, the status quo ought not be changed.)
   b. On an appellate court, on a motion to reverse a lower court decision, a tie sustains the lower court decision. This rule favors the alternative chosen by the lower court. (Rationale: In the absence of a majority to reverse, a lower court decision ought to stand.)

2. *Minority decision rules.* By these rules, if any number larger than a minority of specified size approves, a motion passes; otherwise not. Typically these rules are intended to expedite procedure or to protect minorities. Two examples:
   a. The constitutional requirement that in Congress one-fifth of a house may order the recording of the yeas and nays.
   b. The rule that four judges of the Supreme Court can grant *certiorari* (i.e., they can force the Court to hear a case).

3. *Special majority decision rules.* Under these rules, if a motion receives a specified special majority, say two-thirds, of the votes cast (or possible), it wins; otherwise not. An example of a *relative* special majority is the rule for constitutional amendments in Congress, where the motion to submit an amendment to the states passes with the assent of two-thirds of the votes cast. An example of an *absolute* special majority is the rule that three-fourths of all the states is necessary to ratify an amendment. The motive for such rules is not so much to force decision as it is to protect minorities by supporting the status quo. An extreme form is unanimity, the famous *librum veto* of the Polish Diet, where if $D = (1, 1, \ldots, 1)$, then $F(D) = 1$; otherwise $F(D) = -1$. This rule protects a minority of one.

On the other hand, where a decision is not absolutely necessary and where there is no minority to protect, it is often desirable to use rules that satisfy neutrality. Some of these are:

1. *The jury rule.* As described in section 3.B, unanimity results either in conviction or in acquittal; a tie is a mistrial. The wide range of ties

protects the defendant, and owing to the existence of ties the rule is neutral.

2. *Simple absolute majority decision with no tie-breaking rule.* This method requires that the winner get more than half of the possible votes. If neither alternative does, they tie. This rule is often used on committees such as city councils, to elect a chairperson, and, as is well known, there is often a deadlock when some member abstains.

3. *Rules for more than two alternatives.* This situation is not relevant to the main theme of this chapter, but it does pertain to the condition of neutrality, which must be redefined for the cases of three or more alternatives. Instead of requiring a complete reversal of preference orders, D_i, it is enough to require consistent permutations of the alternatives in them. Then, if the choice from a profile of permuted preference orders is the same as the permutation of the choice from the initial profile, the rule is neutral.[20] The rules discussed in Chapter 2—plurality, Condorcet, Borda, and Bentham rules—all satisfy this neutrality. Although none of them prevents ties, they do guarantee that no alternative has an advantage.

As can be seen from these examples, neutrality is inappropriate when either decisiveness or delay is desired. It is appropriate, however, when one wants to treat alternatives as impartially as possible. In this sense, neutrality is to alternatives what undifferentiatedness is to votes: a technical base for equality. Undifferentiatedness requires that votes be treated equally; neutrality requires that alternatives be treated equally. Hence neutrality is especially appropriate for choice between two candidates, where, in fact, simple majority decision is mainly used, except when tie-breaking is necessary.

Pages 59-64 are omitted.

4

Voting Methods with Three or More Alternatives

Simple majority decision on binary alternatives requires some social embodiment of Procrustes, who chopped off the legs of his guests to fit them into the bed in his inn. The number of alternatives *must* be reduced to exactly two, and this means that some alternatives worthy of consideration must be excised. Furthermore, there must be some Procrustean leader or elite to excise them. Even the apparently unbiased method of reducing alternatives by a series of binary elections requires that someone decide on the *order* of elections—and control of the order is often enough to control the outcome, as we shall see. Thus, however democratic simple majority decision initially appears to be, it cannot in fact be so. Indeed, it is democratic only in the very narrow sense of satisfying certain formal conditions. In any larger sense, it is not democratic because its surrounding institutions must be unfair.

If a voting system is to be really fair, more than two alternatives must be allowed to enter the decision process; a decision method must be able to operate on three or more alternatives. But here is a snag. Many decision methods can deal with several alternatives, but no one method satisfies all the conditions of fairness that have been proposed as reasonable and just. Every method satisfies some and violates others. Unfortunately, there are, so far as I know, no deeper ethical systems nor any deeper axioms for decision that would allow us to judge and choose among these conditions of fairness. Hence there is no generally convincing way to show that one decision method is truly better than another.

So we are faced with a dilemma. Simple majority decision between two alternatives, while narrowly fair, is unattractive because it requires unfair institutions to operate it. On the other hand, no particular decision methods for three or more alternatives can be unequivocally demonstrated to be fair or reasonable. The problem is that we cannot prove that any

method truly and fairly amalgamates the judgments of citizens simply because we do not know what "truly and fairly amalgamates" means.

Chapter 4 is devoted to a demonstration of that proposition. I have no deep preference for any method of decision, and I am not trying to prove that one is better or worse than another. I want merely to show that the notion of true and fair amalgamation is neither obvious nor simple, which is why we cannot easily choose among the voting methods discussed in Chapter 2.

4.A. Some Preliminaries

Because no method for decision among three or more alternatives is entirely satisfactory, many methods have been invented. Despite the variety, they can be classified into three families, the members of which are enough alike that their merits and deficiencies can be discussed together. I call the three families (1) majoritarian methods, (2) positional methods, and (3) utilitarian methods. To begin the discussion I will define the categories and offer some examples.

As a preliminary, however, the vocabulary must be revised and expanded. The set, X, of alternatives now must have $m > 2$ members, $X = (1, 2, \ldots, m)$, which will be referred to with lower-case italic letters, x, y, a, b, and so on. The relations, P, I, and R, remain binary in that they represent individual judgments and votes between a pair of alternatives. But now it must be decided whether they are also transitive—namely, that, if $x\,R_i\,y$ and $y\,R_i\,z$, then $x\,R_i\,z$. (For simplicity, I will write $x\,y\,z$ to mean "$x\,R_i\,y$, $y\,R_i\,z$, and $x\,R_i\,z$," and $x(yz)$ to mean "$x\,P_i\,y$, $y\,I_i\,z$, and $x\,P_i\,z$.") Many arguments are offered for and against individual transitivity.[1] I will, however, conventionally assume that P, R, and I are transitive, except when it is stated otherwise for I and R.

Furthermore, I will also assume that, for all i in the set, N, of decision-makers, some one of these relations connects every pair, x and y, in X. This means that D_i, which is the individual judgment on members of X, is an *ordering*—that is, a transitive and complete arrangement of X. By ordering is meant that the position of each alternative in a particular arrangement is unambiguous. Since X is connected, this means that, for each x and y in X, either xy, yx, or (xy) must be true for each i in N; that is, D_i is one of $\{xy, yx, (xy)\}$. This establishes an unambiguous order in every pair of alternatives. Since D_i is transitive, if it contains $x\,y$ and $y\,z$, then it must contain $x\,z$ (rather than $z\,x$ or $(x\,z)$), which establishes an unambiguous order in every triad of alternatives. And since triads can

overlap (e.g., $w\,x\,y$ and $x\,y\,z$), transitivity establishes an order over an X of any finite size.

The profile, D, will be a set of (not necessarily different) orderings, D_i, of X, for each of n participants; and D will be the set of all possible profiles. Finally, the social choice, F, will now be the result of a decision rule, g, operating on a specified set, X, and a particular profile, D, of judgments to produce a choice: $F_g(X, D)$. Notice that now the set, X, from which the choice is taken will customarily be specified because sometimes the discussion will concern choices from two different sets, X and X'. Typically, if the decision rule, g, is clear from the context, it will not be indicated in the identification of F.

For the time being, except when expressly stated otherwise, I will also assume that all voting is in accordance with preferences. Hence, if $x\,P_i\,y$, person i votes for x over y; and, if $x\,I_i\,y$, person i does not vote for either x or y.

4.B. Majoritarian Methods of Voting

Majoritarian systems are those in which the principles of simple majority voting are extended to three or more alternatives. The rationale is that majority decision is fair and reasonable in its logical structure, and it is assumed that the defect of limiting alternatives can be offset by admitting all desired alternatives. This assumption, however, is profoundly dubious, as we shall see in the discussion of positional voting in section 4.E.

The way of extending the method of majority decision from two to more alternatives is the so-called Condorcet method, in which a winner is defined as that alternative which can beat *all* others in X in a simple majority vote. When X has two members, this is merely simple majority decision. But when X has more than two, this is the requirement that the winner beat $m - 1$ others in $m - 1$ pairwise decisions.

Unfortunately, there are many situations in which the Condorcet winner is undefined simply because no alternative can beat $m - 1$ others. One extreme example, in Display 4-1, is the so-called paradox of voting (see section 1.H). In Display 4-1 each alternative beats one other and loses to another. Which, if any, ought to win?

A more confusing example is set forth in Display 4-2. There, since w ties or beats x, y, and z, presumably w ought to be among the winners. But what of x and y? They tie and so might reasonably win along with w; but although x ties w, y does not, so just as reasonably only w and x

Display 4-1

The Paradox of Voting

D_1:	$x\,y\,z$
D_2:	$y\,z\,x$
D_3:	$z\,x\,y$

Number of Votes for the Alternative in the Row When Placed in Contest Against the Alternative in the Column

	x	y	z
x	—	2	1
y	1	—	2
z	2	1	—

A Condorcet winner would have a majority (here at least 2) in every cell (except the blank one) in a row, signifying that the alternative in the row beats all others. In the paradox, no alternative can beat all others, and so each row has at least one cell with less than a majority (here, at most, 1).

might win. In the absence of a Condorcet winner, problems like these inevitably arise.

Different resolutions of such problems result in different decision methods. Those called *majoritarian* or *Condorcet extensions* have the common property that they select the Condorcet winner, if one exists; if one does not exist, they provide for some further resolution. All such rules depend on knowing whether one alternative beats another in a simple majority vote. Thus I define a *social* relation of majority, M, so that $x\,My$ means "more people prefer x to y than prefer y to x."[2] In any event, to use the Condorcet criterion and its extended rules, it is necessary, as a practical matter, either for each voter to report his or her entire preference structure, D_i, or for the group to hold a number of ballotings, perhaps as many as a round robin, $m(m-1)/2$.

Pages 69-98 are omitted.

4.H. Criteria for Judging Voting Methods

It is clear from the foregoing survey that majoritarian, positional, and utilitarian methods of voting lead to different social results, often strikingly different, from the same underlying individual judgments. It seems natural to wonder, therefore, if there is any way to discriminate among this plethora of devices and to discover a way to amalgamate judgments fairly and truly. There are many criteria for discrimination, and I will discuss some here. Whether they lead to the discovery of a fair and true method I leave to the reader's judgment—though my own belief is that they do not.

I will begin by showing how the properties of simple majority decision discussed in Chapter 3—undifferentiatedness (anonymity), neutrality, and monotonicity—can be generalized to accommodate three or more alternatives. Those three criteria are, in some sense, elementary requirements of fairness and consistency, and most of the voting methods discussed in this chapter satisfy most of them. I will then introduce three deeper requirements of fairness, which embody some of the moral requirements raised in this chapter.

Undifferentiatedness

If the choice from D is x, and if the individual orderings are reassigned among voters to form D', the social choice in D' remains x. Since permuting the preference orders does not change the outcome, it follows that the identity of voters has no effect on the social choice, which is exactly what undifferentiatedness should mean.[25]

Neutrality

If a social profile yields a choice x, and if the elements of X are permuted, thereby creating a new profile, then the choice from the new

profile is the permutation of x. Since a rearrangement of alternatives leads to a corresponding rearrangement of outcomes, it follows that no alternative has a favored position in the voting system, which is exactly what neutrality should mean.[26]

Monotonicity

If a profile changes because some person raises the valuation of x relative to other alternatives, then, if x was originally the social choice, it remains so. Conversely, if a person lowers the valuation of an x that was not originally the social choice, it does not become so. This means that a higher judgment on a winning alternative cannot make it lose; nor can a lower judgment on a loser make it win.[27]

A similar generalization applies to unanimity, as an implication from monotonicity. For unanimity, corresponding to the weak unanimity of equation (N3.6), if, for some alternative, y, all persons prefer some other alternative(s) to it, then y cannot be the social choice. This means that an alternative unanimously beaten by one or more others cannot win.[28]

The Condorcet Criterion

According to the first "deeper" requirement of fairness and consistency, the Condorcet criterion, if an alternative beats (or ties) all others in pairwise contests, then it ought to win.[29] This notion is closely related to the notion of equality and "one man, one vote," in the sense that, when an alternative opposed by a majority wins, quite clearly the votes of some people are not being counted the same as other people's votes.

Consistency

The consistency requirement concerns the way votes are taken from the electorate. If the electorate is divided into two parts for election purposes and if one alternative is chosen in both parts, then it ought to be chosen in the whole.[30]

That this requirement contains a fundamental kind of fairness seems obvious. If a winner is a true winner, subdividing the electorate ought not to make it a loser. If consistency fails to hold, therefore, manipulation is rendered easy. Suppose x wins in the whole electorate and the voting method fails to satisfy consistency. Then opponents of x have merely to set up two appropriately chosen subelectorates, define the winner as that alternative which wins in both, and thereby make y win.

Independence from Irrelevant Alternatives

The independence criterion requires that a method of decision give the same result every time from the same profile of ordinal preferences.[31] This too seems a fundamental requirement of consistency and fairness to prevent the rigging of elections and the unequal treatment of voters; but it has nevertheless been seriously disputed.[32]

Pages 102-114 are omitted.

5

The Meaning of Social Choices

In Chapter 4 I showed that no method of voting could be said to amalgamate individual judgments truly and fairly because every method violates some reasonable canon of fairness and accuracy. All voting methods are therefore in some sense morally imperfect. Furthermore, these imperfect methods can produce different outcomes from the same profile of individual judgments. Hence it follows that sometimes—and usually we never know for sure just when—the social choice is as much an artifact of morally imperfect methods as it is of what people truly want. It is hard to have unbounded confidence in the *justice* of such results.

It is equally hard, as I will show in this chapter, to have unbounded confidence in the *meaning* of such results. Individual persons presumably can, if they think about it deeply enough, order their personal judgments transitively. Hence their valuations mean something, for they clearly indicate a hierarchy of preference that can guide action and choice in a sensible way. But the results of voting do not necessarily have this quality. It is instead the case that *no* method of voting can simultaneously satisfy several elementary conditions of fairness and also produce results that always satisfy elementary conditions of logical arrangement. Hence, not only may the results of voting fail to be fair, they may also fail to make sense. It is the latter possibility that will be analyzed in this chapter.

5.A. Arrow's Theorem

Kenneth Arrow published *Social Choice and Individual Values* in 1951. Although his theorem initially provoked some controversy among economists, its profound political significance was not immediately recog-

nized by political scientists.[1] In the late 1960s, however, a wide variety of philosophers, economists, and political scientists began to appreciate how profoundly unsettling the theorem was and how deeply it called into question some conventionally accepted notions—not only about voting, the subject of this work, but also about the ontological validity of the concept of social welfare, a subject that, fortunately, we can leave to metaphysicians.

The essence of Arrow's theorem is that no method of amalgamating individual judgments can simultaneously satisfy some reasonable conditions of fairness on the method and a condition of logicality on the result. In a sense this theorem is a generalization of the paradox of voting (see section 1.H), for the theorem is the proposition that something like the paradox is possible in *any* fair system of amalgamating values. Thus the theorem is called the *General Possibility Theorem.*

To make the full meaning of Arrow's theorem clear, I will outline the situation and the conditions of fairness and of logicality that cannot simultaneously be satisfied.[2] The situation for amalgamation is:

1. There are n persons, $n \geq 2$, and n is finite. Difficulties comparable to the paradox of voting can arise in individuals who use several standards of judgment for choice. Our concern is, however, *social* choice, so we can ignore the Robinson Crusoe case.

2. There are three or more alternatives—that is, for the set $X = (x_1, \ldots, x_m)$, $m \geq 3$. Since transitivity or other conditions for logical choice are meaningless for fewer than three alternatives and since, indeed, simple majority decision produces a logical result on two alternatives, the conflict between fairness and logicality can only arise when $m \geq 3$.

3. Individuals are able to order the alternatives transitively: If $x R_i y$ and $y R_i z$, then $x R_i z$. If it is not assumed that individuals are able to be logical, then surely it is pointless to expect a group to produce logical results.

The conditions of fairness are:

1. *Universal admissibility of individual orderings (Condition U).* This is the requirement that the set, $\boldsymbol{D}$, includes all possible profiles, D, of individual orders, D_i. If each D_i is some permutation of possible orderings of X by preference and indifference, then this requirement is that individuals can choose any of the possible permutations. For ex-

ample, if $X = (x, y, z)$, the individual may choose any of the following 13 orderings:

1. $x\,y\,z$	7. $x\,(y\,z)$	10. $(x\,y)\,z$	13. $(x\,y\,z)$
2. $y\,z\,x$	8. $y\,(z\,x)$	11. $(y\,z)\,x$	
3. $z\,x\,y$	9. $z\,(x\,y)$	12. $(z\,x)\,y$	
4. $x\,z\,y$			
5. $z\,y\,x$			
6. $y\,x\,z$			(5-1)

The justification for this requirement is straightforward. If social outcomes are to be based exclusively on individual judgments—as seems implicit in any interpretation of democratic methods—then to restrict individual persons' judgments in any way means that the social outcome is based as much on the restriction as it is on individual judgments. Any rule or command that prohibits a person from choosing some preference order is morally unacceptable (or at least unfair) from the point of view of democracy.

2. *Monotonicity.* According to this condition, if a person raises the valuation of a winning alternative, it cannot become a loser; or, if a person lowers the valuation of a losing alternative, it cannot become a winner. The justification for monotonicity was discussed in section 3.B. Given the democratic intention that outcomes be based in some way on participation, it would be the utmost in perversity if the method of choice were to count individual judgments *negatively,* although, as I have shown, some real-world methods actually do so.

3. *Citizens' sovereignty or nonimposition.* Define a social choice as imposed if some alternative, x, is a winner for any set, D, of individual preferences. If x is always chosen, then what individuals want does not have anything to do with social choice. It might, for example, happen that x was everyone's least-liked alternative, yet an imposed choice of x would still select x. In such a situation, voters' judgments have nothing to do with the outcome and democratic participation is meaningless.

4. *Unanimity or Pareto optimality (Condition P).* This is the requirement that, if everyone prefers x to y, then the social choice function, F, does not choose y. (See Chapter 3, note 8, and Chapter 4, note 28.) This is the form in which monotonicity and citizens' sovereignty enter all proofs of Arrow's theorem. There are only two ways that a result contrary to unanimity could occur. One is that the system of amalgamation is not monotonic. Suppose in D' everybody but i prefers x to y

and $y\ P_i'\ x$. Then in D, i changes to $x\ P_i\ y$ so everybody has x preferred to y; but, if F is not monotonic, it may be that x does not belong to $F(\{x, y\}, D)$. The other way a violation of unanimity could occur is for F to impose y even though everybody prefers x to y. Thus the juncture of monotonicity and citizens' sovereignty implies Pareto optimality.

Many writers have interpreted the unanimity condition as purely technical—as, for example, in the discussion of the Schwartz method of completing the Condorcet rule (see section 4.C). But Pareto optimality takes on more force when it is recognized as the carrier of monotonicity and nonimposition, both of which have deep and obvious qualities of fairness.

5. *Independence from irrelevant alternatives (Condition I).* According to this requirement (defined in section 4.H), a method of amalgamation, F, picks the same alternative as the social choice every time F is applied to the same profile, D. Although some writers have regarded this condition simply as a requirement of technical efficiency, it actually has as much moral content as the other fairness conditions (see section 4.H). From the democratic point of view, one wants to base the outcome on the voters' judgments, but doing so is clearly impossible if the method of amalgamation gives different results from identical profiles. This might occur, for example, if choices among alternatives were made by some chance device. Then it is the device, not voters' judgments in D, that determines outcomes. Even if one constructs the device so that the chance of selecting an alternative is proportional in some way to the number of people desiring it (if, for example, two-thirds of the voters prefer x to y, then the device selects x with $p = \frac{2}{3}$), still the expectation is that, of several chance selections, the device will choose x on p selections and y on $1 - p$ selections from the same profile, in clear violation of Condition I. In ancient Greece, election by lot was a useful method for anonymity; today it would be simply a way to by-pass voters' preferences. Another kind of arbitrariness prohibited by the independence condition is utilitarian voting. Based on interpersonal comparisons of distances on scales of unknown length, utilitarian voting gives advantages to persons with finer perception and broader horizons. Furthermore, independence prohibits the arbitrariness of the Borda count (see section 5.F).

6. *Nondictatorship (Condition D).* This is the requirement that there be no person, i, such that, whenever $x\ P_i\ y$, the social choice is x, regardless of the opinions of other persons. Since the whole idea of democracy is to avoid such situations, the moral significance of this condition is obvious.

Finally, the condition of logicality is that the social choice is a weak order, by which is meant that the set, X, is connected and its members can be *socially* ordered by the relation, R, which is the transitive social analogue of preference and indifference combined. (This relation, as in $x\,R\,y$, means that x is chosen over or at least tied with y.) In contrast to the previous discussion, in which the method of amalgamation or choice, F, simply selected an element from X, it is now assumed that F selects repeatedly from pairs in X to produce, by means of successive selections, a social order analogous to the individual orders, D_i. And it is the failure to produce such an order that constitutes a violation of the condition of logicality.[2]

Since an individual weak order or the relation R_i is often spoken of as individual rationality, social transitivity, or R, is sometimes spoken of as collective rationality—Arrow himself so described it. And failure to produce social transitivity can also be regarded as a kind of social irrationality.

Arrow's theorem, then, is that every possible method of amalgamation or choice that satisfies the fairness conditions fails to ensure a social ordering. And if society cannot, with fair methods, be certain to order its outcome, then it is not clear that we can know what the outcomes of a fair method mean. This conclusion appears to be devastating, for it consigns democratic outcomes—and hence the democratic method—to the world of arbitrary nonsense, at least some of the time.

Naturally there has been a variety of attempts to interpret and sidestep this conclusion. One line of inquiry is to raise doubts about its practical importance; another is to look for some theoretical adjustment that deprives the theorem of its force. The rest of this chapter is devoted to a survey of both branches of this huge and important literature, so that in Chapter 6 it will be possible to assess fully the political significance of Arrow's theorem.

I will begin with inquiries about the practical importance of the theorem. One such inquiry is an estimate of the expected frequency of profiles, D, that do not lead to a transitive order.

5.B. The Practical Relevance of Arrow's Theorem: The Frequency of Cycles

One meaning of Arrow's theorem is that, under any system of voting or amalgamation, instances of intransitive or cyclical outcomes can occur.

Since, by definition, no one of the alternatives in a cycle can beat all the others, there is no Condorcet winner among cycled alternatives. All cycled alternatives tie with respect to their position in a social arrangement in the sense that $x\ y\ z\ x$, $y\ z\ x\ y$, and $z\ x\ y\ z$ have equal claims to being the social arrangement. Borda voting similarly produces a direct tie among cycled alternatives. Hence a social arrangement is indeterminate when a cycle exists. When the arrangement is indeterminate, the actual choice is arbitrarily made. The selection is not determined by the preference of the voters. Rather it is determined by the power of some chooser to dominate the choice or to manipulate the process to his or her advantage. Every cycle thus represents the failure of the voting process. One way to inquire into the practical significance of Arrow's theorem is, therefore, to estimate how often cycles can occur.

For this estimate, a number of simplifying assumptions are necessary. For one thing, majority voting (rather than positional voting or any other kind of amalgamation) is always assumed. This assumption of course limits the interpretation severely. For another thing, only cycles that preclude a Condorcet winner are of interest. Voting may fail to produce a weak order in several ways:

1. With all three alternatives, there may be a cycle: $x\ R\ y\ R\ z\ R\ x$ or simply $x\ y\ z\ x$.
2. With four or more alternatives, there may be
   a. A Condorcet winner followed by a cycle: $w\ x\ y\ z\ x$
   b. A cycle among all alternatives: $w\ x\ y\ z\ w$; or intersecting cycles: $s\ t\ w\ x\ y\ z\ w\ v\ s$
   c. A cycle in which all members beat some other alternative: $x\ y\ z\ x\ w$

If one is interested in social welfare judgments involving an ordering of all alternatives, then all cycles are significant no matter where they occur. But if one is interested in picking out a social choice, as in the voting mechanisms discussed here, then the significant cases are only 1, 2(b), and 2(c), where there is no unique social choice. (These are often called *top cycles*.) Attempts to estimate the significance of Arrow's theorem by some sort of calculation have all been made from the point of view of social choice rather than welfare judgments and have therefore concerned the frequency of top cycles.

For Arrow's theorem, Condition U allows individuals to have any weak ordering, R_i, of preference and indifference, as in (5.1). Calculation is simpler, however, based on strong orders—that is, individual preference orders, P_i, with indifference not allowed.

With m alternatives, there are $m!$ (i.e., $1 \cdot 2 \cdot \ldots \cdot m$) such linear orders possible; and, when $m = 3$, these are:

$$x\,y\,z, \quad x\,z\,y, \quad y\,x\,z, \quad y\,z\,x, \quad z\,x\,y, \quad z\,y\,x$$

Each such order is a potential D_i. When each of n voters picks some (not necessarily different) D_i, a profile, D, is created. Since the first voter picks from $m!$ orders, the second from $m!, \ldots,$ and the last from $m!$, the number of possible different profiles, D, is $(m!)^n$, which is the number of members of the set, $\boldsymbol{D}$, of all profiles, when voters have only strong orders.

A calculation that yields some estimate of the significance of cycles is the fraction, $p(n, m)$, of D in $\boldsymbol{D}$ without a Condorcet winner:

$$p(n, m) = \frac{\text{Number of } D \text{ without a Condorcet winner}}{(m!)^n}$$

If one assumes that each D is equally likely to occur (which implies also that, for each voter, the chance of picking some order is $1/m!$), then $p(n, m)$ is an a priori estimate of the probability of the occurrence of a top cycle. Several calculations have been made, as set forth in Display 5-1.[3] As is apparent from the Display, as the number of voters and alternatives increases, so do the number of profiles without a Condorcet winner. The calculation thereby implies that instances of the paradox of voting are very common. Most social choices are made from many alternatives (though often we do not realize this fact because the number has been winnowed down by various devices such as primary elections and committees that select alternatives for agendas) and by many people, so the calculations imply that Condorcet winners do not exist in almost all decisions.

But, of course, there are a number of reasons to believe that such calculations are meaningless. People do not choose an ordering with probability $1/m!$. Rather, at any particular moment, some orders are more likely to be chosen than others. The six strong orders over triples generate two cycles:

"Forward Cycle"	"Backward Cycle"
1. $x\,y\,z$	4. $x\,z\,y$
2. $y\,z\,x$	5. $z\,y\,x$
3. $z\,x\,y$	6. $y\,x\,z$

(5-2)

Display 5-1

Values of $p(n, m)$: Proportion of Possible Profiles Without a Condorcet Winner

	n = Number of Voters						
m = Number of Alternatives	3	5	7	9	11	...	Limit
3	.056	.069	.075	.078	.080		.088
4	.111	.139	.150	.156	.160		.176
5	.160	.200	.215				.251
6	.202						.315
Limit	1.000	1.000	1.000	1.000	1.000		1.000

The entry in the row for four alternatives and in the column for seven voters—namely, .150—is the ratio of the number of profiles without a Condorcet winner to the number of profiles possible when seven voters order four alternatives.

Cycles occur when voters concentrate on one or the other of these sets of three orders. But suppose voters are induced by, for example, political parties, to concentrate heavily on, say, (1), (2), and (5). Then there is no cycle. Furthermore, there is good reason to believe that debate and discussion do lead to such fundamental similarities of judgment. Calculations based on equiprobable choices very likely seriously overestimate the frequency of cycles in the natural world.

On the other hand, it is clear that one way to manipulate outcomes is to generate a cycle. Suppose that in Display 5-2 profile D exists and that person 2 realizes that his or her first choice, y, will lose to the Condorcet winner, x. Person 2 can at least prevent that outcome by generating a cycle (or a tie) by voting as if his or her preference were $y\ z\ x$ as in D'.

The tendency toward similarity may thus reduce the number $p(n, m)$, while the possibility of manipulation may increase the number. It seems to me that similarity probably reduces the number of profiles without Condorcet winners on issues that are not very important and that no one has a motive to manipulate, while the possibility of manipulation

Display 5-2

The Generation of a Cycle

D	
D_1:	$x\ y\ z$
D_2:	$y\ x\ z$
D_3:	$z\ x\ y$

Note. Majoritarian ordering of D: $x\ P\ y\ P\ z$.

D'	
D'_1:	$x\ y\ z$
D'_2:	$y\ z\ x$
D'_3:	$z\ x\ y$

Note. Cycle in D' under majoritarian voting: $x\ P\ y\ P\ z\ P\ x$.

In D' person 2 has reversed z and x from D, thereby generating a cycle.

increases the number of such profiles on important issues, where the outcome is worth the time and effort of prospective losers to generate a top cycle. Neither of these influences appears in the calculations and thus renders them suspect from two opposite points of view.