INFM 718A / LBSC 705
Information For Decision Making

Lecture 8

Overview

- Conditional probabilities
  - Review textbook example
- Decision trees
  - Textbook example
- In-Class exercises
  - Conditional probabilities – 4.3
  - Decision trees – 5.1
- Assignment 4 preview

Joint Probability

- Probability that two independent events will occur together.
- E.g.: Throw a dice, flip a coin, what is the probability that the die shows 1 and the coin shows heads. P (1 and H) = 1/6 * 1/2 =1/12

Joint Probability

- General formula:
  $P(A \cap B) = P(A|B)*P(B)$
- Independent events:
  $P(A \cap B) = P(A)*P(B)$

Conditional Probability

- Probability of an outcome, under the condition that another, dependent event has had a certain outcome. E.g.: Probability that the die shows 1, based on the information that it shows an odd number. P (1 | odd).

Bayes' Theorem

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(B \cap A)}{P(B \cap A) + P(B \cap \overline{A})}$$

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid \overline{A}) \times P(\overline{A})}$$

Bayes' Theorem

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

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$$P(A \mid B) \times P(B) = P(A \cap B) = P(B \mid A) \times P(A)$$

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

In-Class Exercise 4.3

- Data:
  - P(U) = 0.02 → P(!U) = 0.98
  - P(+|U) = 0.90 → P(-|U) = 0.10
  - P(-| !U) = 0.99 → P(+| !U) = 0.01
- First, calculate joint probabilities
- Then, calculate posterior probabilities (reverse conditional probabilities)

Joint Probabilities

- $P(+\cap U) = P(+|U)*P(U)$
  $= 0.90*0.02 = 0.018$
- $P(-\cap U) = P(-|U)*P(U)$
  $= 0.10*0.02 = 0.002$
- $P(+\cap !U) = P(+|!U)*P(!U)$
  $= 0.01*0.98 = 0.0098$
- $P(-\cap !U) = P(-|!U)*P(!U)$
  $= 0.99*0.98 = 0.9702$

Sums of Joint Probabilities

- $P(+) = P(+\cap U) + P(+\cap !U)$
  $= 0.018 + 0.0098 = 0.0278$
- $P(-) = P(-\cap U) + P(-\cap !U)$
  $= 0.002 + 0.9702 = 0.9722$

Reverse Conditional Probabilities

- $P(U|+) = P(U\cap +)/P(+)$
  $= 0.018/0.0278 = 0.6475$
- $P(U|-) = P(U\cap -)/P(-)$
  $= 0.0098/0.0278 = 0.3525$
- $P(!U|+) = P(!U\cap +)/P(+)$
  $= 0.002/0.9722 = 0.0021$
- $P(!U|-) = P(!U\cap -)/P(-)$
  $= 0.9702/0.9722 = 0.9979$

Interpretation

- Probability that the person is actually a user when you get a positive result is 64.75%
- Probability that the person is **not** a user when you get a positive result is 35.25%

Decision Trees

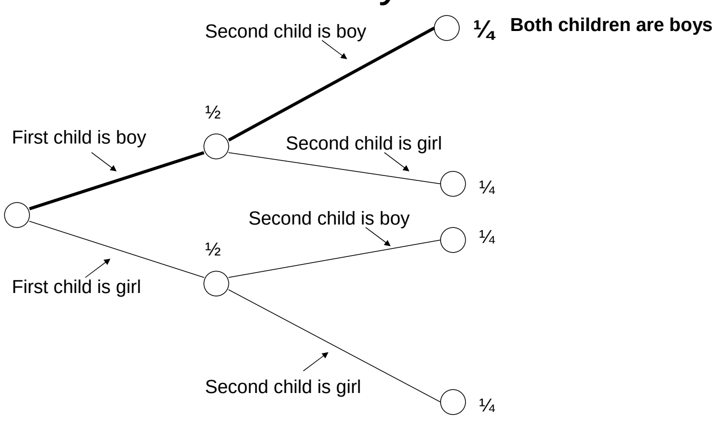
- The main difference between decision trees and probability trees:
  - Probability trees have only (chance) event nodes, and no decision nodes.
  - Decision trees have decision nodes in addition to event nodes.

Probability Trees

- A type of visual model to represent joint probabilities.

- E.g.: A woman has two children. What is the probability that both children are boys?

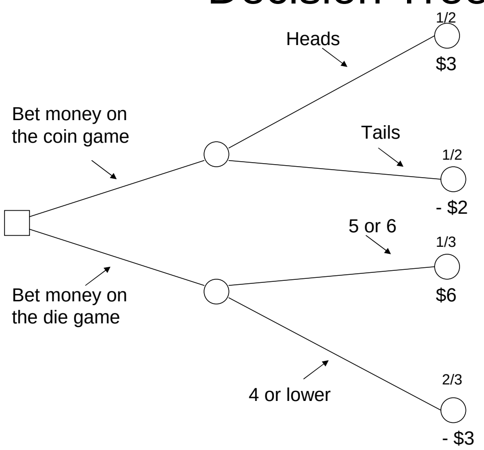
Probability Trees



Decision Trees

- E.g.: We will play a game. You decide whether to have a coin flipped or a die tossed.
  - You need to bet $2 for the coin game. If it's heads, you will get $3, if it's tails, you will lose the $2.
  - You need to bet $3 for the die game. If you toss 5 or 6, you will get $6, if you toss 4 or lower, you will lose the $3.

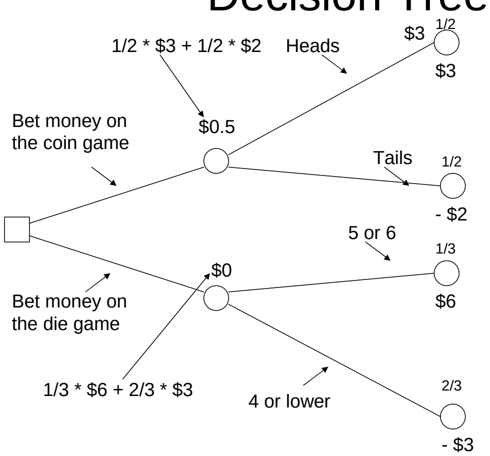
Decision Trees



Expected Value

- Sum of products (SUMPRODUCT) of probabilities and payoffs of all possible outcomes of a chance event, or a decision.
- E (Coin game) = 1/2 * $3 + 1/2 * $2 = $0.5
- E (Die game) = 1/3 * $6 + 2/3 * $3 = $0
- It is a better strategy to play the coin game.

Decision Trees



Decision Trees

- Textbook example (pp. 284-297)
  – Medfly eradication
- Install TreePlan
  – BlackBoard > Course Documents > Add-Ins
  – Download and unzip the file
  – Open treeplan.xla in Excel

In-class Exercises

- Conditional probabilities:
  - In-class Exercise 4.3
- Decision trees:
  - In-class Exercise 5.1

- Assignment 4 preview